Eureka Math™ Homework Helper

2015–2016

Grade 6
Module 1
Lessons 1–29

G6-M1-Lesson 1: Ratios

1.   At the local movie theatre, there are 115 boys, 92 girls, and 28 adults.

   a.   Write the ratio of the number of boys to the number of girls.

   115 : 92

   b.   Write the same ratio using another form ($A : B$ vs. A to B).

   115 to 92

   c.   Write the ratio of the number of boys to the number of adults.

   115 : 28

   d.   Write the same ratio using another form.

   115 to 28

> I know that I can represent a ratio using a colon or the word "to."

2.   At a restaurant, 120 bottles of water are placed in ice at the buffet. At the end of the dinner rush, 36 bottles of water remained.

   a.   What is the ratio of the number of bottles of water taken to the total number of water bottles?

   84 to 120, *or* 84 : 120

   b.   What is the ratio of the number of water bottles remaining to the number of water bottles taken?

   36 to 84, *or* 36 : 84

> I need to subtract the number of water bottles remaining from the total number of water bottles to determine the number of water bottles taken.

3.   Choose a situation that could be described by the following ratios, and write a sentence to describe the ratio in the context of the situation you chose.

   a.   1 to 3

   For every one yard, there are three feet.

   b.   7 to 30

   For every 7 days in a week, often there are 30 days in a month.
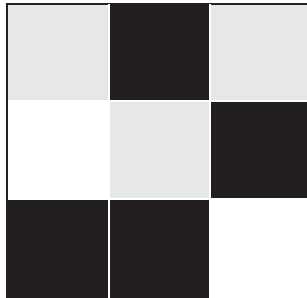
   c.   26 : 6

   For every 26 weeks, there are typically 6 months.

> I should choose situations that make sense with the numbers in the ratios. I know that for every one yard, there are three feet.

G6-M1-Lesson 2: Ratios

Examples

1. Using the design below, create 4 different ratios related to the image. Describe the ratio relationship, and write the ratio in the form $A:B$ or the form A to B.

> I see that there are 2 white tiles, 3 grey tiles, and 4 black tiles. I also see that there are 9 tiles altogether. I can use these quantities, the words "for each," "for every," or "to." I can also use a colon.

For every 9 tiles, there are 4 black tiles.

The ratio of the number of black tiles to the number of white tiles is 4 to 2.

The ratio of the number of grey tiles to the number of white tiles is 3 : 2.

There are 2 black tiles for each white tile.

Answers will vary.

2. Jaime wrote the ratio of the number of oranges to the number of pears as 2 : 3. Did Jaime write the correct ratio? Why or why not?

> I see that there are 3 oranges and 2 pears. I also know that the first value in the ratio relationship is the number of oranges, so that number is represented first in the ratio. The number of pears comes second in the relationship, so that number is represented second in the ratio.

Jaime is incorrect. There are three oranges and two pears.
The ratio of the number of oranges to the number of
pears is 3 : 2.

G6-M1-Lesson 3: Equivalent Ratios

1. Write two ratios that are equivalent to $2:2$.

    $2 \times 2 = 4, 2 \times 2 = 4$; *therefore, an equivalent ratio is* $4:4$.

    $2 \times 3 = 6, 2 \times 3 = 6$; *therefore, an equivalent ratio is* $6:6$.
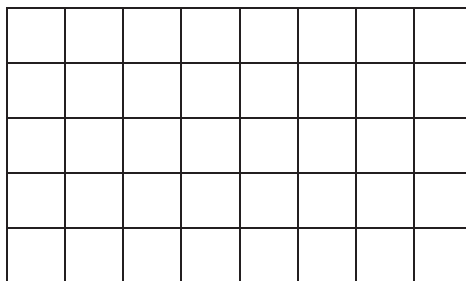
    Answers will vary.

> The ratio is in the form $A:B$. I must multiply the A and B values by the same nonzero number to determine equivalent ratios.

2. Write two ratios that are equivalent to $5:13$.

    $5 \times 2 = 10, 13 \times 2 = 26$; *therefore, an equivalent ratio is* $10:26$.

    $5 \times 4 = 20, 13 \times 4 = 52$; *therefore, an equivalent ratio is* $20:52$.

3. The ratio of the length of the rectangle to the width of the rectangle is _____ to _____.

> The length of this rectangle is 8 units, and the width is 5 units. Because the value for the length is listed first in the relationship, 8 is first in the ratio (or the A value). 5 is the B value.

The ratio of the length of the rectangle to the width of the rectangle is $8:5$.

4. For a project in health class, Kaylee and Mike record the number of pints of water they drink each day. Kaylee drinks 3 pints of water each day, and Mike drinks 2 pints of water each day.

   a. Write a ratio of the number of pints of water Kaylee drinks to the number of pints of water Mike drinks each day.

   $3:2$

   b. Represent this scenario with tape diagrams.

   Number of pints of water Kaylee drinks

   Number of pints of water Mike drinks

   c. If one pint of water is equivalent to 2 cups of water, how many cups of water did Kaylee and Mike each drink? How do you know?

   Kaylee drinks 6 cups of water because $3 \times 2 = 6$. Mike drinks 4 cups of water because $2 \times 2 = 4$. Since each pint represents 2 cups, I multiplied the number of pints of water Kaylee drinks by two and the number of pints of water Mike drinks by two. Also, since each unit represents two cups:

   Number of pints of water Kaylee drinks | 2 | 2 | 2 |

   Number of pints of water Mike drinks | 2 | 2 |

   > Each unit in the tape diagrams represents 2 because there are two cups for every pint of water.

   d. Write a ratio of the number of cups of water Kaylee drinks to the number of cups of water Mike drinks.

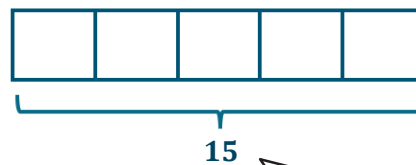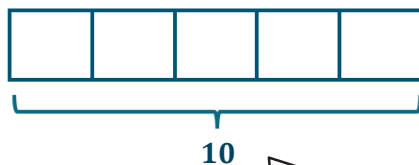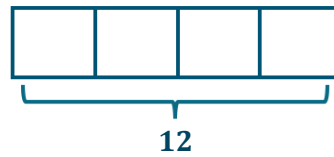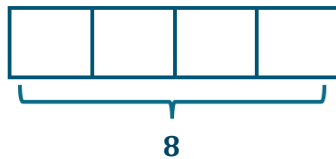   *The ratio of the number of cups of water Kaylee drinks to the number of cups of water Mike drinks is $6:4$.*

   e. Are the two ratios you determined equivalent? Explain why or why not.

   $3:2$ and $6:4$ are equivalent because they represent the same value. The diagrams never changed, only the value of each unit in the diagram.

G6-M1-Lesson 4: Equivalent Ratios

1. Use diagrams or the description of equivalent ratios to show that the ratios $4:5$, $8:10$, and $12:15$ are equivalent.

8

12

> Each tape diagram represents the ratio $4:5$. In each diagram, there are four units in the first tape, and five units in the second tape.

10

15

> Eight is two times four. Ten is two times five.

> Twelve is three times four. Fifteen is three times five.

The constant number, c, is 2*.*   *The constant number, c, is* 3*.*

2. The ratio of the amount of John's money to the amount of Rick's money is $5:13$. If John has $25, how much money do Rick and John have together? Use diagrams to illustrate your answer.

John's Money

$$25 \div 5 = 5$$

Each unit represents 5*.*
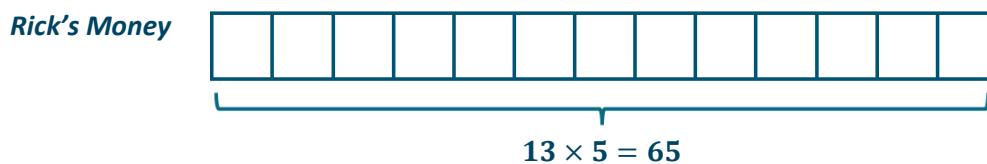
> Five units in the tape diagram represents John's portion of the $5:13$ ratio. Thirteen units represents Rick's portion of the ratio.

25

Rick's Money

$$13 \times 5 = 65$$

5 *units represents* 25*. That means that* 1 *unit represents* 5*. Since all of the units are the same,* 13 *units represents* 65 *because* $13 \times 5 = 65$*. To determine how much money John and Rick have together, add the amounts.* $25 + 65 = 90$*.*

G6-M1-Lesson 5: Solving Problems by Finding Equivalent Ratios

1. The ratio of the number of females at a spring concert to the number of males is $7:3$. There are a total of 450 females and males at the concert. How many males are in attendance? How many females?

Number of Females

Number of Males

450
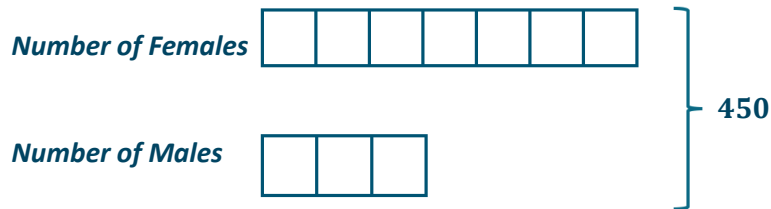
I know that there are a total of ten equal units. To determine the value of one unit, I need to divide 450 by 10. Each unit represents 45.

10 *units* $\rightarrow 450$

1 *unit* $\rightarrow 450 \div 10 = 45$

3 *units* $\rightarrow 45 \times 3 = 135$
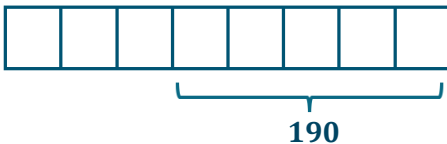
7 *units* $\rightarrow 45 \times 7 = 315$

Because there are 3 units that represent the number of males, I need to multiply each unit by 3. $45 \times 3 = 135$.

Because there are 7 units that represent the number of females, I need to multiply each unit by 7. $45 \times 7 = 315$.

2. The ratio of the number of adults to the number of students at a field trip has to be $3:8$. During a current field trip, there are 190 more students on the trip than there are adults. How many students are attending the field trip? How many adults?

Number of Adults

Number of Students

190

> 5 equal units represent the 190 more students than adults. To determine the value of one unit, I need to divide 190 by 5. Each unit represents 38.

3×38
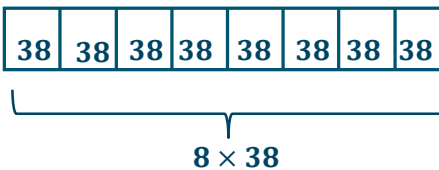
5 units → 190

1 unit → 190 ÷ 5 = 38

3 units → 3 × 38 = 114

8 units → 8 × 38 = 304

Number of Adults | 38 | 38 | 38 |

Number of Students | 38 | 38 | 38 | 38 | 38 | 38 | 38 | 38 |

8×38

There are 304 students and 114 adults attending the field trip.

G6-M1-Lesson 6: Solving Problems by Finding Equivalent Ratios

Solving Ratio Problems

At the beginning of Grade 6, the ratio of the number of students who chose art as their favorite subject to the number of students who chose science as their favorite subject was $4:9$. However, with the addition of an exciting new art program, some students changed their mind, and after voting again, the ratio of the number of students who chose art as their favorite subject to the number of students who chose science as their favorite subject changed to $6:7$. After voting again, there were 84 students who chose art as their favorite subject. How many fewer students chose science as their favorite subject after the addition of the new art program than before the addition of the new art program? Explain.

<u>*Before the New Art Program*</u>

Chose Art

Chose Science

<u>*After the New Art Program*</u>

Chose Art

Chose Science
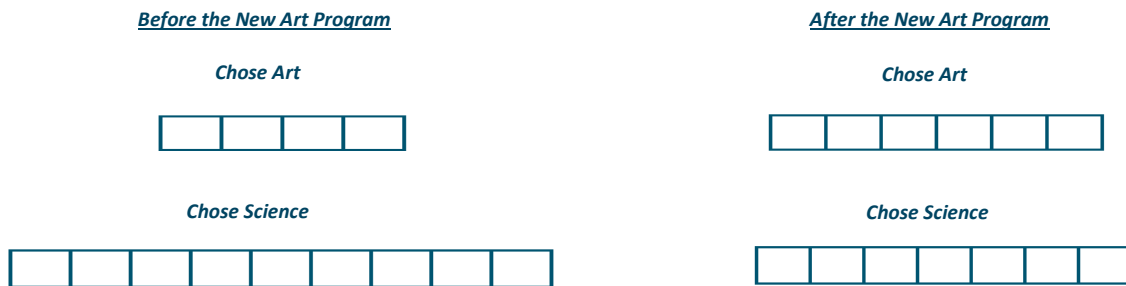
6 *units* → 84

1 *unit* → 84 ÷ 6 = 14

9 *units* → 14 · 9 = 126

7 *units* → 14 · 7 = 98

126 − 98 = 28

> I can draw and label tape diagrams to represent each ratio. If 84 students chose art after the new art program, then 6 units represent a value of 84, so 1 unit has a value of 14 (84 divided by 6). This information will allow me to determine the value of 9 units and 7 units. Now I'm able to find the difference and answer the question.

There were 28 fewer students who chose science as their favorite subject after the addition of the new art program than the number of students who chose science as their favorite subject before the addition of the new art program. 126 students chose science before, and 98 students chose science after the new art program was added.

G6-M1-Lesson 7: Associated Ratios and the Value of a Ratio

1. Amy is making cheese omelets for her family for breakfast to surprise them. For every 2 eggs, she needs $\frac{1}{2}$ cup of cheddar cheese. To have enough eggs for all the omelets she is making, she calculated she would need 16 eggs. If there are 5.5 cups of cheddar cheese in the fridge, does Amy have enough cheese to make the omelets?   Why or why not?

- $2: \frac{1}{2}$

- *Value of the Ratio:* **4**

    $2: \frac{1}{2}$ $16: 4$

    2 *is four times as much as* $\frac{1}{2}$.

    16 *is four times as much as* 4.

    Amy needs 4 *cups of cheddar cheese.* *She will have enough cheese because she needs* 4 *cups and has* 5.5 *cups.*

> I need to determine the value of the ratio in order to find the amount of cheese that is needed. I can do this by dividing 2 by $\frac{1}{2}$. The number of cups of cheese needed is $\frac{1}{4}$ the number of eggs. I can also say the number of eggs is 4 times the number of cups of cheese.

2. Samantha is a part of the Drama Team at school and needs pink paint for a prop they're creating for the upcoming school play. Unfortunately, the 6 gallons of pink paint she bought is too dark. After researching how to lighten the paint to make the color she needs, she found out that she can mix $\frac{1}{3}$ of a gallon of white paint with 2 gallons of the pink paint she bought. How many gallons of white paint will Samantha have to buy to lighten the 6 gallons of pink paint?

- $\frac{1}{3} : 2$

- *Value of the Ratio:* $\frac{1}{6}$

$\frac{1}{3}$ *is* $\frac{1}{6}$ *of* 2; 1 *is* $\frac{1}{6}$ *of* 6

Samantha would need 1 *gallon of white paint to make the shade of pink she desires.*

> I need to determine the value of the ratio by dividing $\frac{1}{3}$ by 2. The number of gallons of white paint needed is $\frac{1}{6}$ of the number of gallons of pink paint. I can also say the number of gallons of pink paint is 6 times the number of gallons of white paint.

G6-M1-Lesson 8: Equivalent Ratios Defined Through the Value of a Ratio

1. Use the value of the ratio to determine which ratios are equivalent to $9:22$.

   a. $10:23$

   b. $27:66$

   c. $22.5:55$

   d. $4.5:11$

   Answer choices (b), (c), and (d) are equivalent to $9:22$.

   > I can divide 9 by 22 in order to find the value of the ratio, which is $\frac{9}{22}$. To find the value of the ratio for all the answer choices, I need to divide:
   >
   > $$\frac{10}{23}$$
   > $$\frac{27}{66} = \frac{9}{22}$$
   > $$\frac{22.5}{55} = \frac{9}{22}$$
   > $$\frac{4.5}{11} = \frac{9}{22}$$

2. The ratio of the number of shaded sections to unshaded sections is $3:5$. What is the value of the ratio of the number of shaded sections to the number of unshaded sections?

   $\frac{3}{5}$

   > To find the value of the ratio, I divide 3 by 5. The value of the ratio is $\frac{3}{5}$.

3. The middle school band has 600 members. $\frac{1}{5}$ of the members were chosen for the highly selective All-State Band. What is the value of the ratio of the number of students who were chosen for the All-State Band to the number of students who were not chosen for the All-State Band?

   Students chosen for the All-State Band

   Students not chosen for the All-State Band
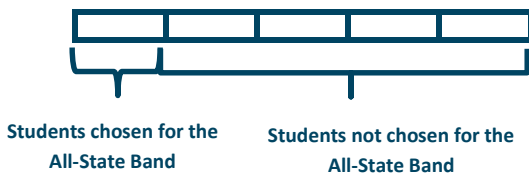
   The value of the ratio of the number of students who were chosen for the All-State Band to the number of students who were not chosen for the All-State Band is $\frac{1}{4}$.

   > In the tape diagram, $\frac{1}{5}$ of the members in the band were chosen for the All-State Band. I can divide 600 by 5, which is 120, so I know 120 students were selected for the All-State Band and 480 (600 – 120) were not. I also know the value of one unit, which is 120, and the value of 4 units, which is 480. The value of the ratio is $\frac{120}{480}$ or $\frac{1}{4}$.

4. Tina is learning to juggle and has set a personal goal of juggling for at least five seconds. She tried 30 times but only accomplished her goal 14 times.

    a. Describe and write more than one ratio related to this situation.

    The ratio of the number of successful tries to the total number of tries is $14:30$*.*

    The ratio of the number of successful tries to the number of unsuccessful tries is $14:16$*.*

    The ratio of the number of unsuccessful tries to the number of successful tries is $16:14$*.*

    The ratio of the number of unsuccessful tries to the total number of tries is $16:30$*.*

> There is more than one ratio associated with this problem. I know the total, 30, and the number of times she was successful, 14. I can also determine the number of times she was unsuccessful $(30 - 14 = 16)$.

    b. For each ratio you created, use the value of the ratio to express one quantity as a fraction of the other quantity.

    The number of successful tries is $\frac{14}{30}$ *or* $\frac{7}{15}$ *of the total number of tries.*

    The number of successful tries is $\frac{14}{16}$ *or* $\frac{7}{8}$ *the number of unsuccessful tries.*

    The number of unsuccessful tries is $\frac{16}{14}$ *or* $\frac{8}{7}$ *the number of successful tries.*

    The number of unsuccessful tries is $\frac{16}{30}$ *or* $\frac{8}{15}$ *of the total number of tries.*

    c. Create a word problem that a student can solve using one of the ratios and its value.

    If Tina tries juggling for at least five seconds 15 *times, how many successes would she anticipate having, assuming her ratio of successful tries to unsuccessful tries does not change?*

G6-M1-Lesson 9: Tables of Equivalent Ratios

Assume the following represents a table of equivalent ratios. Fill in the missing values. Then create a real-world context for the ratios shown in the table.

6	**13**
12	26
18	39
24	**52**
30	65
36	**78**

I need to find the value of the ratio for $18\colon 39$ and $30\colon 65$ (they should be the same since they are equivalent ratios). I can divide 18 by 39 and 30 by 65. The value of the ratio is $\frac{6}{13}$.

Sample Answer: Brianna is mixing red and white paint to make a particular shade of pink paint. For every 6 tablespoons of white paint, she mixes 13 tablespoons of red paint. How many tablespoons of red paint would she need for 30 tablespoons of white paint?

G6-M1-Lesson 10: The Structure of Ratio Tables—Additive and Multiplicative

1. Lenard made a table to show how much blue and yellow paint he needs to mix to reach the shade of green he will use to paint the ramps at the skate park. He wants to use the table to make larger and smaller batches of green paint.

Blue	Yellow
10	4
15	6
20	8
25	10

> I see that the value in the first column keeps increasing by 5, and the value in the second column keeps increasing by 2, so the ratio is $5:2$. All of the ratios listed in the table are equivalent.

a. What ratio was used to create this table? Support your answer.

The ratio of the amount of blue paint to the amount of yellow paint is $5:2$. $10:4$, $15:6$, $20:8$, and $25:10$ are all equivalent to $5:2$.

b. How are the values in each row related to each other?

In each row, the amount of yellow paint is $\frac{2}{5}$ the amount of blue paint, or the amount of blue paint is $\frac{5}{2}$ the amount of yellow paint.

c. How are the values in each column related to each other?

The values in the columns are increasing using the ratio. Since the ratio of the amount of blue paint to the amount of yellow paint is $5:2$, I repeatedly added to form the table. 5 was added to the entries in the blue column, and 2 was added to the entries in the yellow column.

2.

    a. Create a ratio table for making 2-ingredient banana pancakes with a banana-to-egg ratio of $1:2$.
        Show how many eggs would be needed to make banana pancakes if you use 14 bananas.

Number of Bananas	Number of Eggs
1	2
2	4
3	6
4	8
14	28

> I need to label the missing title: Number of Eggs. I can complete the table using the relationship: for every 1 banana, I need 2 eggs. I can add 1 repeatedly in the first column and add 2 repeatedly in the second column to determine values in the table. Or, I can multiply the values in the first column by two because the number of eggs is twice the number of bananas.

      28 *eggs would be needed to make banana pancakes if* 14 *bananas are used.*

    b. How is the value of the ratio used to create the table?

      The value of the ratio of the number of bananas to the number of eggs is $\frac{1}{2}$. If I know the number of bananas, I can multiply that amount by 2 to get the number of eggs. If I know the number of eggs, I can multiply that amount by $\frac{1}{2}$ (or divide by 2) to get the number of bananas.

G6-M1-Lesson 11: Comparing Ratios Using Ratio Tables

1. Jasmine and Juliet were texting.

   a. Use the ratio tables below to determine who texts the fastest.

      Jasmine

Time (min)	2	5	6	8
Words	56	140	168	224

      Juliet

Time (min)	3	4	7	10
Words	99	132	231	330

      > If Jasmine can text 56 words in 2 minutes, I can determine how many words she can text in 1 minute by dividing both numbers by 2.

      > If Juliet can text 99 words in 3 minutes, I can determine how many words she texts in 1 minute by dividing both numbers by 3.

      Juliet texts the fastest because she texts **33** *words in* **1** *minute, which is faster than Jasmine who texts* **28** *words in* **1** *minute.*

   b. Explain the method that you used to determine your answer.

      To determine how many words Jasmine texts in a minute, I divided **56** *by* **2** *since she texted* **56** *words in* **2** *minutes. So, Jasmine texts* **28** *words in* **1** *minute. For Juliet, I divided* **99** *by* **3** *since she texted* **99** *words in* **3** *minutes. So, Juliet texts* **33** *words in* **1** *minute.*

2. Victor is making lemonade. His first recipe calls for 2 cups of water and the juice from 12 lemons. His second recipe says he will need 3 cups of water and the juice from 15 lemons. Use ratio tables to determine which lemonade recipe calls for more lemons compared to water.

Recipe 1   *Recipe 2*

Water (cups)	2	4	6
Lemons	12	24	36

Water (cups)	3	6	9
Lemons	15	30	45

For every 2 cups of water, Victor will use the juice from 12 lemons. Using this ratio 2: 12, I can create equivalent ratios in the table.

For every 3 cups of water, Victor will use the juice from 15 lemons. Using this ratio 3: 15, I can create equivalent ratios in the table.

Now that I have determined a few equivalent ratios for each table, I can compare the number of lemons needed for 6 cups of water since 6 cups of water is a value in each of the tables. I notice for Recipe 1, I need 6 more lemons for the same number of cups of water.

Recipe 1 uses more lemons compared to water. When comparing 6 cups of water, there were more lemons used in Recipe 1 than in Recipe 2.

G6-M1-Lesson 12: From Ratio Tables to Double Number Line Diagrams

1. David earns $6 an hour for helping with yard work. He wants to buy a new video game that costs $27. How many hours must he help in the yard to earn $27 to buy the game? Use a double number line diagram to support your answer.

> Since 27 is midway between 24 and 30, I can find the corresponding quantity by finding the midway between 4 and 5.

| 1 | 2 | 3 | 4 | 4.5 | 5 | **Hours Worked** |

Money Earned

| 6 | 12 | 18 | 24 | 27 | 30 | *(in dollars)* |

> For every 1 hour, David makes 6 dollars.

> 6 is not a factor of 27, but it is a factor of 24 and 30. 27 is midway between 24 and 30.
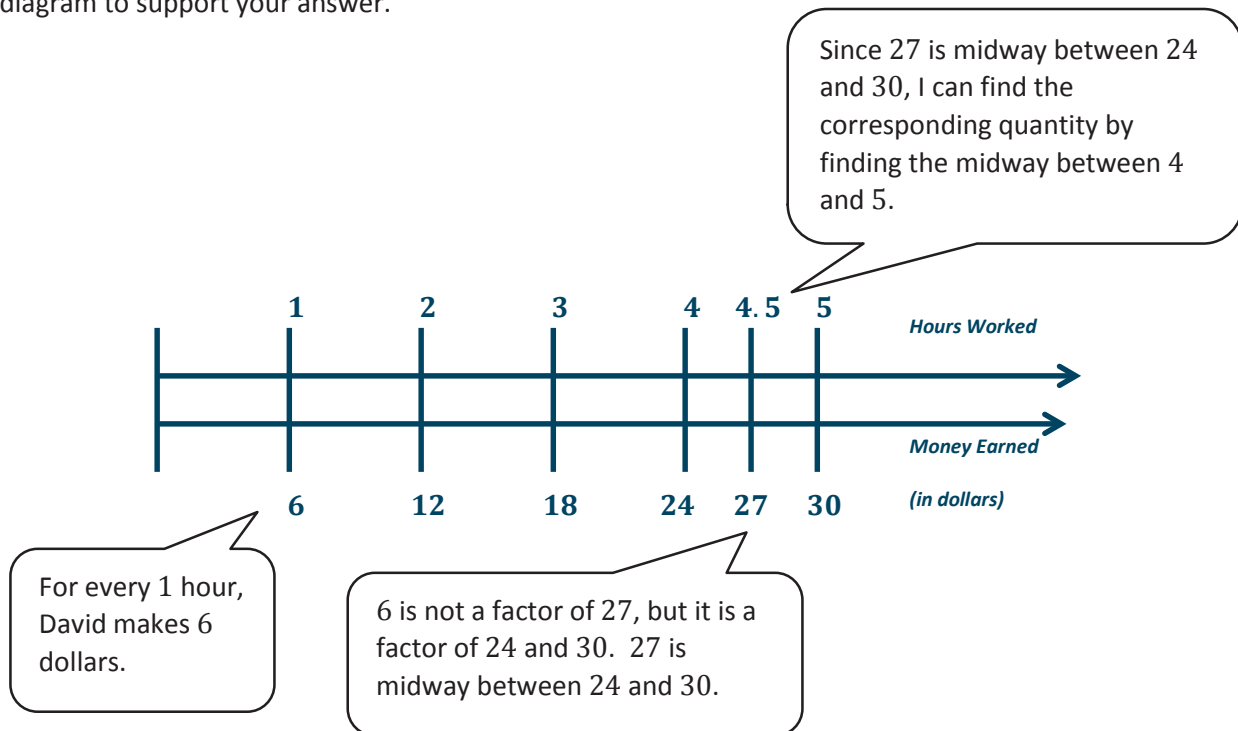
David will earn $27 after working for 4.5 hours.

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2. During migration, a duck flies at a constant rate for 11 hours, during which time he travels 550 miles. The duck must travel another 250 miles in order to reach his destination. If the duck maintains the same constant speed, how long will it take him to complete the remaining 250 miles? Include a table or diagram to support your answer.

> The duck flew at a constant rate for 11 hours and traveled 550 miles.

	1	5	11		**Hours**

| | 50 | 250 | 550 | | **Miles** |

> I can divide both 11 and 550 by 11 to determine how many miles flown in 1 hour.

> I can multiply both 1 and 50 by 5 to reach 250 miles.
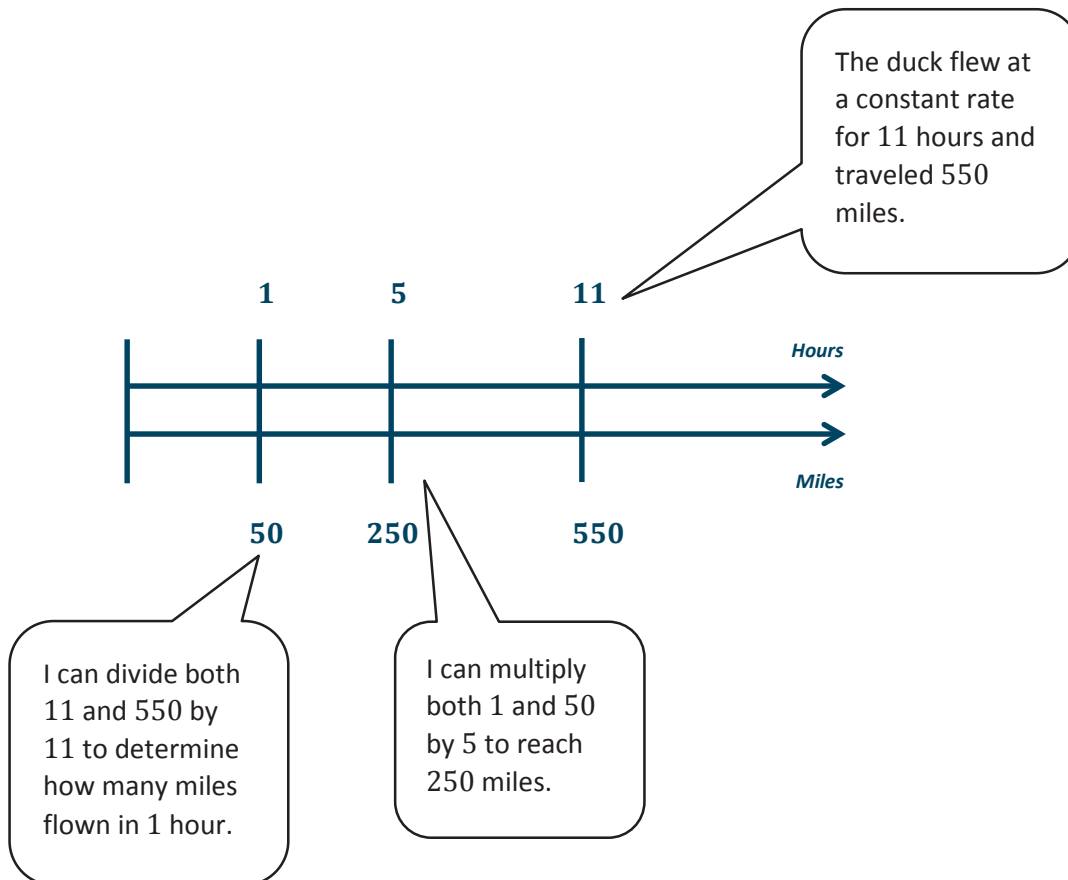
It will take the duck 5 hours to travel the remaining 250 miles.

G6-M1-Lesson 13: From Ratio Tables to Equations Using the Value of a Ratio

A pie recipe calls for 2 teaspoons of cinnamon and 3 teaspoons of nutmeg.

Make a table showing the comparison of the number of teaspoons of cinnamon and the number of teaspoons of nutmeg.

Number of Teaspoons of Cinnamon (C)	Number of Teaspoons of Nutmeg (N)
2	3
4	6
6	9
8	12
10	15

> I know the ratio of teaspoons of cinnamon to teaspoons of nutmeg is 2: 3 because that information is given in the problem. I will write this ratio in the first row of the table and then determine equivalent ratios.

1. Write the value of the ratio of the number of teaspoons of cinnamon to the number of teaspoons of nutmeg.

$$\frac{2}{3}$$

> Anytime I see a ratio relationship, I pay close attention to the order. In this problem, I'm comparing the number of teaspoons of cinnamon to the number of teaspoons of nutmeg. So, I look at the first row in my table. The numerator is the number of teaspoons of cinnamon, which is 2, and the denominator is the number of teaspoons of nutmeg, which is 3.

2. Write an equation that shows the relationship of the number of teaspoons of cinnamon to the number of teaspoons of nutmeg.

$$N = \frac{3}{2}C \text{ or } C = \frac{2}{3}N$$

> To write an equation, I have to pay close attention to the value of the ratio for teaspoons of nutmeg to teaspoons of cinnamon, which is $\frac{3}{2}$. Now, I can write the equation $N = \frac{3}{2}C$.

2. Explain how the value of the ratio of the number of teaspoons of nutmeg to the number of teaspoons of cinnamon can be seen in the table.

The values in the first row show the values in the ratio. The ratio of the number of teaspoons of nutmeg to the number of teaspoons of cinnamon is $3:2$. The value of the ratio is $\frac{3}{2}$.

3. Explain how the value of the ratio of the number of teaspoons of nutmeg to the number of teaspoons of cinnamon can be seen in an equation.

The number of teaspoons of nutmeg is represented as N in the equation. The number of teaspoons of cinnamon is represented as C. The value of the ratio is represented because the number of teaspoons of nutmeg is $\frac{3}{2}$ times as much as the number of teaspoons of cinnamon, $N = \frac{3}{2}C$.

4. Using the same recipe, compare the number of teaspoons of cinnamon to the number of total teaspoons of spices used in the recipe.

    Make a table showing the comparison of the number of teaspoons of cinnamon to the number of total teaspoons of spices.

Number of Teaspoons of Cinnamon (C)	Total Number of Teaspoons of Spices (T)
2	5
4	10
6	15
8	20
10	25

> To get the total, I will look at the table I made on the previous page. I see that for every 2 teaspoons of cinnamon, I will need 3 teaspoons of nutmeg, so for 2 teaspoons of cinnamon, there are 5 total teaspoons of spices since $2 + 3 = 5$.

5. Write the value of the ratio of the amount of total teaspoons of spices to the number of teaspoons of cinnamon.

$\dfrac{5}{2}$

> I will look at the first row. There are 5 total teaspoons of spices and 2 teaspoons of cinnamon. Now I can write the value of the ratio.

6. Write an equation that shows the relationship of total teaspoons of spices to the number of teaspoons of cinnamon.

$T = \dfrac{5}{2}C$

> To write this equation, I will use the value of the ratio that I determined from Problem 5, which is $\dfrac{5}{2}$.

G6-M1-Lesson 14: From Ratio Tables, Equations, and Double Number Line Diagrams to Plots on the Coordinate Plane

1. Write a story context that would be represented by the ratio 1: 7.

   Answers will vary. Example: For every hour Sami rakes leaves, she earns $7.

   > I can think of a situation that compares 1 of one quantity to 7 of another quantity. For every 1 hour she rakes leaves, Sami earns $7.

   Complete a table of values and graph.

| Number of Hours Spent Raking Leaves	Amount of Money Earned in Dollars
1	7
2	14
3	21
4	28
5	35

Pay for Raking Leaves
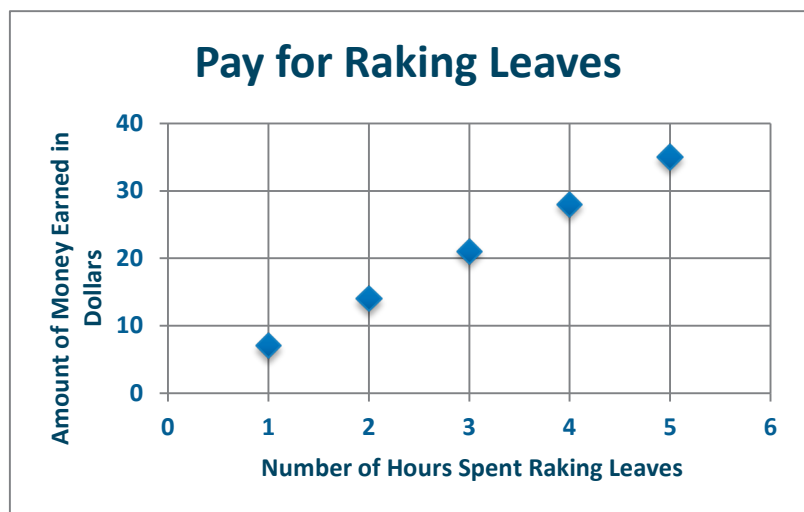
2. Complete the table of values to find the following:

Find the number of cups of strawberries needed if for every jar of jam Sarah makes, she has to use 5 cups of strawberries.

Number of Jars of Jam	Number of Cups of Strawberries
1	5
2	10
3	15
4	20
5	25

> I can start with the ratio I know from the problem. For every 1 jar of jam, Sarah uses 5 cups of strawberries, so the ratio is 1: 5, and I will write this ratio in the first row of my table. I can use this information to determine equivalent ratios.
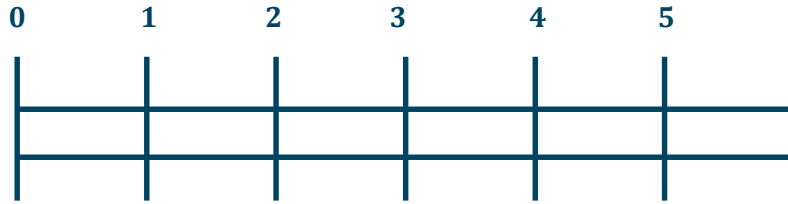
Use a graph to represent the relationship.
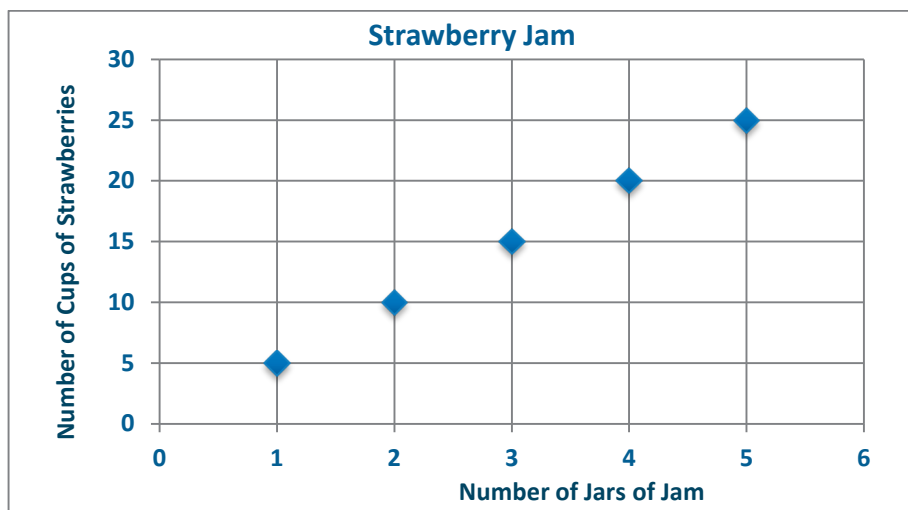
Create a double number line diagram to show the relationship.

**Number of
Jars of Jam**

| 0 | 1 | 2 | 3 | 4 | 5 |

**Number of
Cups of
Strawberries**

| 0 | 5 | 10 | 15 | 20 | 25 |

To create the double number line diagram, I can use the ratio 1 to 5 and the equivalent ratios I listed in my table.

G6-M1-Lesson 15: A Synthesis of Representations of Equivalent Ratio Collections

1. When the video of Tillman the Skateboarding Bulldog was first posted, it had 300 views after 4 hours. Create a table to show how many views the video would have after the first, second, and third hours after posting, if the video receives views at the same rate. How many views would the video receive after 5 hours?

Number of Hours	Number of Views
1	75
2	150
3	225
4	300
5	375

First, I can record the information I know in the table. I know there were 300 views after 4 hours. I will determine how many views there were after 1 hour by dividing 300 by 4, which is 75, so there were 75 views after 1 hour. Knowing there were 75 views in 1 hour will allow me to figure out how many views there were after 2 hours, 3 hours, and 5 hours by multiplying the number of hours by 75.

After five hours, the video would have 375 *views.*

2. Write an equation that represents the relationship from Problem 1. Do you see any connections between the equation you wrote and the ratio of the number of views to the number of hours?

$v = 75 \ h$ *The constant in the equation,* 75, *is the number of views after* 1 *hour.*

To write the equation, I determine variables to represent hours and views. I can choose v to represent views and h to represent hours. Since the number of views is dependent on how many hours passed since the video was posted, views is the dependent variable and hours is the independent variable. To find out the number of views, I will multiply the hours by 75, the constant.

3. Use the table in Problem 1 to make a list of ordered pairs that you could plot on a coordinate plane.
   $(1, 75), (2, 150), (3, 225), (4, 300), (5, 375)$

4. Graph the ordered pairs on a coordinate plane. Label your axes, and create a title for the graph.

Video Views

(x-axis: Number of Hours Since Video Was Posted; y-axis: Number of Views)
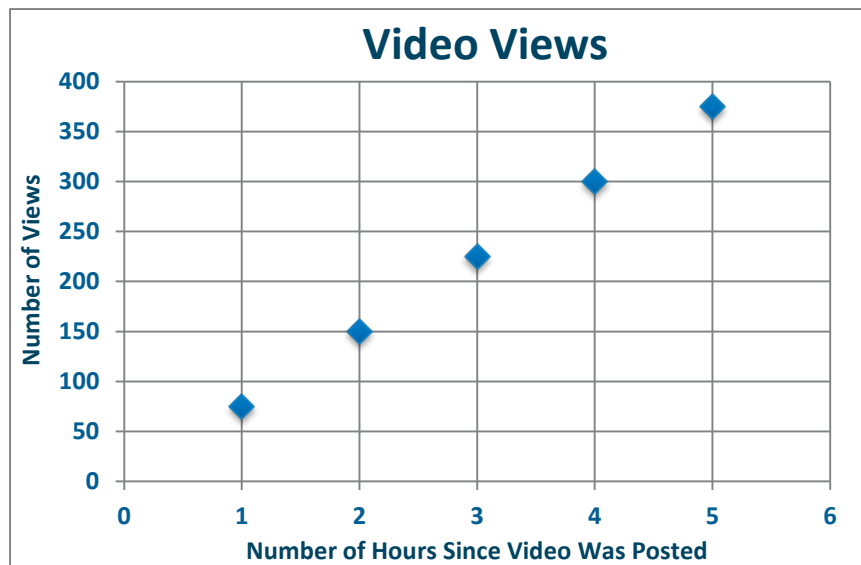
5. Use multiple tools to predict how many views the website would have after 15 hours.
   Answers may vary but could include all representations from the module. The correct answer is $1,125$ views.

   - *If the equation $v = 75\,h$ is used, multiply 75 by the number of hours, which is 15. So,*
     $$75 \times 15 = 1,125.$$
   - *To determine the answer using the table, extend the table to show the number of views after 15 hours.*
   - *To use the graph, extend the x-axis to show 15 hours, and then plot the points for 7 through 15 hours since those values are not currently on the graph.*
   - *In a tape diagram, 1 unit has a value of 75 since there were 75 views after the video was posted for 1 hour, so 15 units has a value of $75 \times 15 = 1,125$.*

G6-M1-Lesson 16: From Ratios to Rates

1. The Canter family is downsizing and saving money when they grocery shop. In order to do that, they need to know how to find better prices. At the grocery store downtown, grapes cost $2.55 for 2 lb., and at the farmer's market, grapes cost $3.55 for 3 lb.

    a.   What is the unit price of grapes at each store? If necessary, round to the nearest penny.

Grocery Store

Number of Pounds of Grapes	1	2
Cost (in dollars)	1.28	2.55

Farmer's Market

Number of Pounds of Grapes	1	3
Cost (in dollars)	1.18	3.55

> I know the price of two pounds. To find the price of one pound, I need to divide the cost by two. $2.55 divided by 2 is $1.275. I need to round to the nearest penny, so the price of one pound is $1.28.

The unit price for the grapes at the grocery store is 1.28*.*

The unit price for the grapes at the farmer's market is 1.18*.*

    b.   If the Canter family wants to save money, where should they purchase grapes?

The Canter family should purchase the grapes from the farmer's market. Their unit price is lower, so they pay less money per pound than if they would purchase grapes from the grocery store downtown.

2. Oranges are on sale at the grocery store downtown and at the farmer's market. At the grocery store, a 4 lb. bag of oranges cost $4.99, and at the farmer's market, the price for a 10 lb. bag of oranges is $11.99. Which store offers the best deal on oranges? How do you know? How much better is the deal?

Grocery Store

Number of Pounds of Oranges	1	4
Cost (in dollars)	1.25	4.99

> Now I see that I can divide the cost by the number of pounds to determine the unit price, or the cost of one pound.

Farmer's Market

Number of Pounds of Oranges	1	10
Cost (in dollars)	1.20	11.99

The unit price for the oranges at the grocery store is $1.25. *The unit price for the oranges at the farmer's market is* $1.20. *The farmer's market offers a better deal on the oranges. Their price is* $0.05 *cheaper per pound than the grocery store.*

> To determine how much better the deal is, I need to subtract the smallest unit price from the greatest unit price to find the difference. $1.25 − $1.20 = $0.05. The farmer's market's unit price is five cents cheaper than the grocery store's unit price.

G6-M1-Lesson 17: From Rates to Ratios

Examples

1. An express train travels at a cruising rate of $150 \frac{\text{miles}}{\text{hour}}$. If the train travels at this average speed for 6 hours, how far does the train travel while at this speed?

Number of Miles	150	300	450	600	750	900
Number of Hours	1	2	3	4	5	6

I know the ratio of the number of miles to the number of hours is $150:1$. I can create equivalent ratios in a ratio table to determine how many miles the train will travel in 6 hours. $1 \times 6 = 6$, so $150 \times 6 = 900$.

The train will travel **900** *miles in 6 hours traveling at this average cruising speed.*

2. The average amount of rainfall in Baltimore, Maryland in the month of April is $\frac{1}{10} \frac{\text{inch}}{\text{day}}$. Using this rate, how many inches of rain does Baltimore receive on average for the month of April?
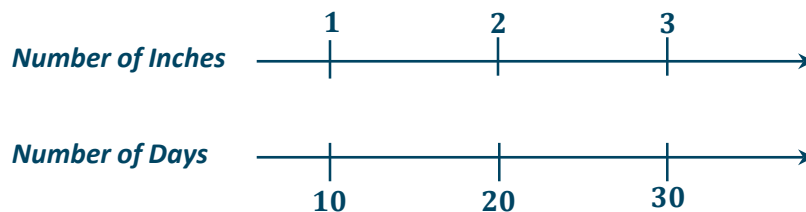
I know the ratio of the number of inches to the number of days is $1:10$. I can create a double number line diagram to determine equivalent ratios. $10 \times 3 = 30$, so $1 \times 3 = 3$.

At this rate, Baltimore receives **3** *inches of rain in the month of April.*

G6-M1-Lesson 18: Finding a Rate by Dividing Two Quantities

1. Ami earns $15 per hour working at the local greenhouse. If she worked 13 hours this month, how much money did she make this month?

$$\frac{15 \text{ dollars}}{1 \text{ hour}} \cdot 13 \text{ hours} = 15 \text{ dollars} \cdot 13 = 195 \text{ dollars}$$

I know the rate of Ami's pay is 15 dollars for every 1 hour, or $15 \frac{\text{dollars}}{\text{hour}}$. I can multiply the amount of hours by this rate to determine the amount of dollars she makes this month.

At a rate of $15 \frac{\text{dollars}}{\text{hour}}$*, Ami will make* $195 *if she works* 13 *hours.*

2. Trisha is filling her pool. Her pool holds 18,000 gallons of water. The hose she is filling the pool with pumps water at a rate of $300 \frac{\text{gallons}}{\text{hour}}$. If she wants to open her pool in 72 hours, will the pool be full in time?

$$300 \frac{\text{gallons}}{\text{hour}} \cdot 72 \text{ hours} = 300 \text{ gallons} \cdot 72 = 21,600 \text{ gallons}$$

Trisha has plenty of time to fill her pool at this rate. It takes 72 *hours to fill* 21,600 *gallons. She only needs to fill* 18,000 *gallons.*

G6-M1-Lesson 19: Comparison Shopping – Unit Price and Related Measurement Conversions

1. Luke is deciding which motorcycle he would like to purchase from the dealership. He has two favorites and will base his final decision on which has the better gas efficiency (the motorcycle that provides more miles for every gallon of gas). The data he received about his first choice, the Trifecta, is represented in the table. The data he received about his second choice, the Zephyr, is represented in the graph. Which motorcycle should Luke purchase?
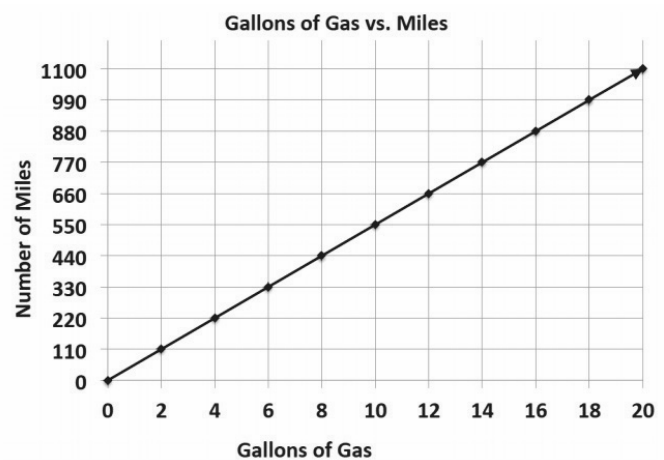
Trifecta:

Gallons of Gas	3	6	9
Number of Miles	180	360	540

> To determine the unit rate, I need to find out how many miles the motorcycle will travel using one gallon of gas. To do that, I can divide each of these values by 3.

Zephyr:

> One gallon of gas is not represented on the graph. To determine unit rate, I can look at the relationship of 110 miles for every 2 gallons of gas. I can divide each of those quantities by two to determine the unit rate.

The Trifecta gets $60 \frac{\text{miles}}{\text{gallon}}$ *because* $\frac{180}{3} = 60$. *The Zephyr gets* $55 \frac{\text{miles}}{\text{gallon}}$ *because* $\frac{110}{2} = 55$. *Luke should purchase the Trifecta because it gets more miles for every gallon of gas.*

2. Just as Luke made his final decision, the dealer suggested purchasing the Comet, which gets 928 miles for every tank fill up. The gas tank holds 16 gallons of gas. Is the Comet Luke's best choice, based on miles per gallon?

$$\frac{928}{16}\ \frac{\text{miles}}{\text{gallon}} = 58\ \frac{\text{miles}}{\text{gallon}}$$

Luke should still purchase the Trifecta. The Comet only provides 58 *miles per gallon, which is less than the* 60 *miles per gallon the Trifecta provides.*

G6-M1-Lesson 20: Comparison Shopping – Unit Price and Related Measurement Conversions

1. The table below shows how much money Hillary makes working at a yogurt shop. How much money does Hillary make per hour?

Number of Hours Worked	2	4	6	8
Money Earned (in dollars)	25.50	51	76.50	102

To determine the unit rate, I need to find out how much money Hillary makes in one hour. Since I know how much money she makes in 2 hours, I can divide both of these values by 2.

Hillary earns $\frac{25.50}{2} \frac{dollars}{hour}$. $25.50 \div 2 = 12.75$. Hillary earns $\$12.75$ per hour.

2. Makenna is also an employee at the yogurt shop. She earns $2.00 more an hour than Hillary. Complete the table below to show the amount of money Makenna earns.

Number of Hours Worked	3	6	9	12
Money Earned (in dollars)	**44.25**	**88.50**	**132.75**	**177**

$$14.75 \; \frac{dollars}{\cancel{hour}} \cdot 3 \; \cancel{hours} = 44.25 \text{ dollars}$$

$$14.75 \; \frac{dollars}{\cancel{hour}} \cdot 6 \; \cancel{hours} = 88.50 \text{ dollars}$$

$$14.75 \; \frac{dollars}{\cancel{hour}} \cdot 9 \; \cancel{hours} = 132.75 \text{ dollars}$$

$$14.75 \; \frac{dollars}{\cancel{hour}} \cdot 12 \; \cancel{hours} = 177 \text{ dollars}$$

3. Colbie is also an employee of the yogurt shop. The amount of money she earns is represented by the equation $m = 15h$, where h represents the number of hours worked and m represents the amount of money she earns in dollars. How much more money does Colbie earn an hour than Hillary? Explain your thinking.

    The amount of money that Colbie earns for every hour is represented by the constant 15. This tells me that Colbie earns 15 dollars per hour. To determine how much more money an hour she earns than Hillary, I need to subtract Hillary's pay rate from Colbie's pay rate. $15 - 12.75 = 2.25$. Colbie makes 2.25 more dollars per hour than Hillary.

4. Makenna recently received a raise and now makes the same amount of money per hour as Colbie. How much more money per hour does Makenna make now, after her promotion? Explain your thinking.

    Makenna now earns the same amount of money per hour as Colbie, which is $15 \frac{\text{dollars}}{\text{hour}}$. She previously earned $14.75 \frac{\text{dollars}}{\text{hour}}$. To determine how much more money Makenna makes now, after her promotion, I need to subtract her previous pay rate from her current pay rate. $15 - 14.75 = 0.25$. Makenna makes 0.25 dollars more an hour, or she makes 25 cents more an hour.
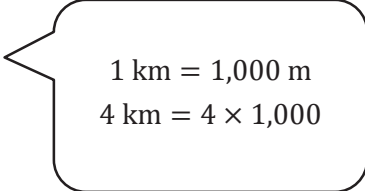
G6-M1-Lesson 21: Getting the Job Done – Speed, Work, and Measurement Units

Note:

Students should have the conversion chart supplied to them for this assignment.
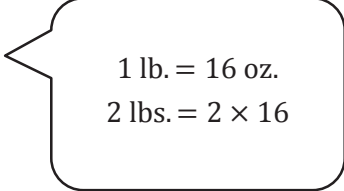
1. 4 km = _____ m

    4,000 m

    > $1 \text{ km} = 1{,}000 \text{ m}$
    > $4 \text{ km} = 4 \times 1{,}000$

2. Matt buys 2 pounds of popcorn. He will give each friend a one ounce bag of popcorn. How many bags can Matt make?

    32 *bags*

    > $1 \text{ lb.} = 16 \text{ oz.}$
    > $2 \text{ lbs.} = 2 \times 16$

G6-M1-Lesson 22: Getting the Job Done – Speed, Work, and Measurement Units

1. A biplane travels at a constant speed of $500 \frac{\text{kilometers}}{\text{hour}}$. It travels at this rate for 2 hours. How far did the biplane travel in this time?

$$500 \; \frac{\textbf{kilometers}}{\textbf{hour}} \cdot \textbf{2 hours} = \textbf{500 kilometers} \cdot \textbf{2} = \textbf{1,000 kilometers}$$

> The distance formula is $d = r \cdot t$,
> distance $=$ rate $\times$ time

2. Tina ran a 50 yard race in 5.5 seconds. What is her rate of speed?

$$\frac{\textbf{50}}{\textbf{5.5}} \; \frac{\textbf{yards}}{\textbf{second}} = \textbf{9.09} \; \frac{\textbf{yards}}{\textbf{second}}$$

> $r = \frac{d}{t}$

2015-16

G6-M1-Lesson 23: Problem Solving Using Rates, Unit Rates, and Conversions

Who runs at a faster rate: someone who runs 40 yards in 5.8 seconds or someone who runs 100 yards in 10 seconds?

$$\frac{40}{5.8} \frac{\text{yards}}{\text{second}} \approx 6.9 \frac{\text{yards}}{\text{second}}$$

$$\frac{100}{10} \frac{\text{yards}}{\text{second}} \approx 10 \frac{\text{yards}}{\text{second}} \rightarrow \textit{faster}$$

> Find the unit rate by dividing. Compare the unit rates to determine the fastest runner.

G6-M1-Lesson 24: Percent and Rates per 100

1. Holly owns a home cleaning service. Her company, Holly N'Helpers, which consists of three employees, has 100 homes to clean this month. Use the 10×10 grid to model how the work could have been distributed between the three employees. Using your model, complete the table.

   Answers can vary as students choose how they want to separate the workload. This is a sample response.

B	B	G	G	G	G	G	P	P	P
B	B	G	G	G	G	G	P	P	P
B	B	G	G	G	G	G	P	P	P
B	B	G	G	G	G	G	P	P	P
B	B	G	G	G	G	G	P	P	P
B	B	B	G	G	G	G	P	P	P
B	B	B	G	G	G	G	P	P	P
B	B	B	G	G	G	G	P	P	P
B	B	B	G	G	G	G	P	P	P
B	B	B	G	G	G	G	P	P	P

Worker	Percentage	Fraction	Decimal
Employee (B)	25%	$\dfrac{25}{100}$	0.25
Employee (G)	45%	$\dfrac{45}{100}$	0.45
Employee (P)	30%	$\dfrac{30}{100}$	0.30

I will assign Employee (B) 25 houses to clean, Employee (G) 45 houses to clean, and Employee (P) 30 houses to clean.

I know percents are out of a total of 100 and are another way to show a part–to–whole ratio. Since there are 100 houses to clean, the total is 100 in this example. Since Employee B is assigned to 25 homes, the ratio is $25:100$, the fraction is $\frac{25}{100}$, the decimal is 0.25, and the percentage is 25%. Using this reasoning, I was able to complete the table for Employees G and P.

2. When hosting Math Carnival at the middle school, 80 percent of the budget is spent on prizes for the winners of each game. Shade the grid below to represent the portion of the budget that is spent on prizes.

0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01

> This is a 10x10 grid, so I know there are 100 total blocks. I know each block represents $\frac{1}{100}$, 0.01, or 1%. 80% means 80 out of 100, so I will shade 80 blocks.

a. What does each block represent?

Each block represents $\frac{1}{100}$ of the total budget.

> Because there are 100 total blocks, which represents the entire budget, each block represents $\frac{1}{100}$ of the total budget.
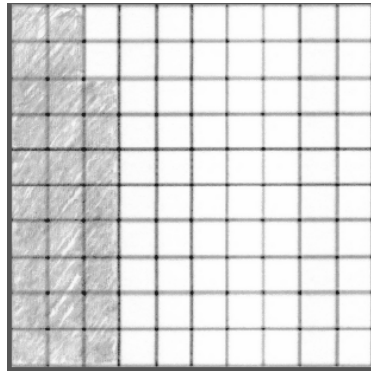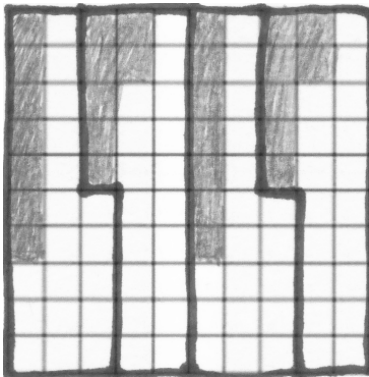
b. What percent of the budget was not spent on prizes?

20%

> I know from the problem that 80% of the total budget was spent on prizes, so this leaves 20% of the budget for other expenses. $100\% - 80\% = 20\%$

G6-M1-Lesson 25: A Fraction as a Percent

1. Use the 10×10 grid to express the fraction $\frac{7}{25}$ as a percent.

Students should shade 28 *of the squares in the grid.*

> There are 100 squares in the grid. Percent is a part–to–whole comparison where the whole is 100. The fraction is $\frac{7}{25}$, so I will divide the whole (100 squares) into 4 parts since $100 \div 25 = 4$. I can shade 7 squares in each part as seen in the first grid because the fraction $\frac{7}{25}$ tells me there are 7 shaded squares for every group of 25. Since there are 4 groups of 25 in 100, I can also multiply 7 by 4, which will give me the total number of shaded squares, 28.

2. Use a **tape diagram** to relate the fraction $\frac{7}{25}$ to a **percent**.

0	7	14	21	28

0	25	50	75	100
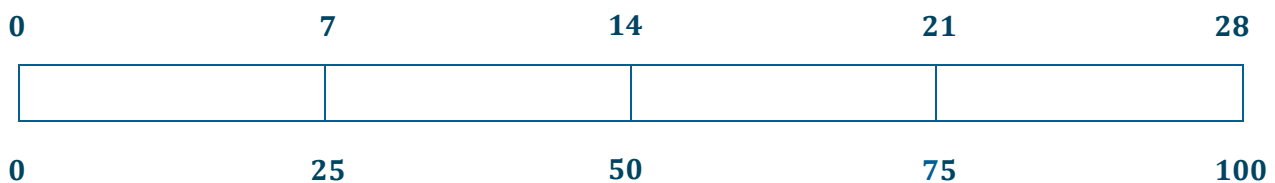
3. How are the diagrams related?
    Both show that $\frac{7}{25}$ *is the same as* $\frac{28}{100}$.

> Both grids show that $\frac{7}{25}$ is equal to $\frac{28}{100}$. The tape diagram also shows the fractions are the same.

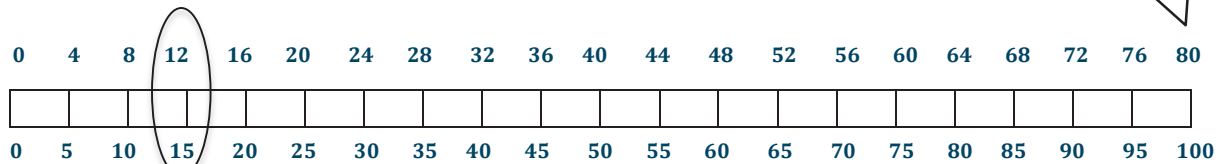4. What **decimal** is also related to the **fraction**?

   0.28

5. Which diagram is the most helpful for converting the fraction to a decimal? Explain why.

   Answers will vary according to student preferences. Possible student response: It is most helpful to use the tape diagram for converting the fraction to a decimal. To convert $\frac{7}{25}$ to a decimal, I can clearly see how there are 4 groups of 25 in 100. So, to find 4 groups of 7, I can multiply 4×7, which is 28. I know $\frac{7}{25}$ is equal to $\frac{28}{100}$, which is 0.28.

G6-M1-Lesson 26: Percent of a Quantity

1. What is 15% of 80? Create a model to prove your answer.

> I know the whole, 100%, is 80.

| 0	4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76	80
0	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100

15% of 80 is 12.

> First, I will draw a model (tape diagram). Since the whole is 80, I know 80 is equal to 100%. I can determine that 50% is 40 because $80 \div 2 = 40$. I can also determine 10% by dividing 80 by 10 since $100\% \div 10 = 10\%$. $80 \div 10 = 8$, so 10% of 80 is 8. By knowing 10% is 8, I can determine what 5% is by dividing by 2. I know 5% of 80 is 4. I can continue labeling the values on my tape diagram, counting up by 5's on the bottom and counting up by 4's on the top. I know 15% of 80 is 12.

2. If 30% of a number is 84, what was the original number?

> We know from the problem that 30% of a number is 84, so I will label this on the tape diagram.

> This is the value I will be determining.

0	28	56	84	112	140	168	196	224	252	280
0	10	20	30	40	50	60	70	80	90	100
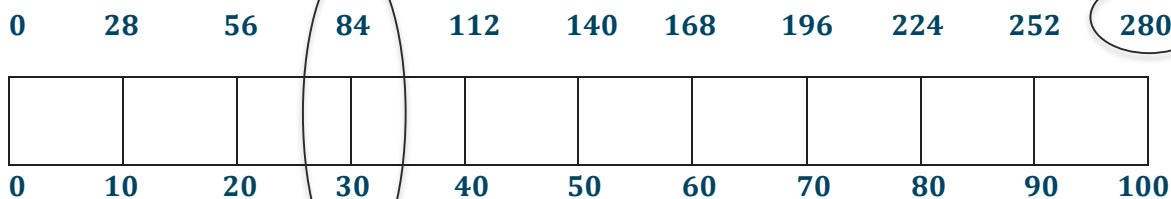
> Because the whole represents 100%, I can divide the tape diagram into 10 parts so each part represents 10%. I know 30% is 84. Since I know 30% is 84, I can find 10% by dividing 84 by 3 since $30\% \div 3 = 10\%$. $84 \div 3 = 28$, so 10% is 28. Now that I know the value of 10%, I can determine the value of 100% by multiplying $28 \times 10 = 280$. So, the value of the whole (the original number) is 280.

3. In a 10×10 grid that represents 500, one square represents ___**5**___.

   Use the grid below to represent 17% of 500.
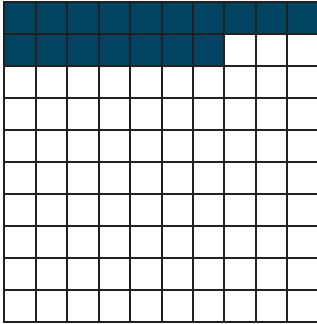
> Since this whole grid represents 500, each square has a value of 5. There are 17 squares shaded in, so $17 \times 5 = 85$. So, 17% of 500 is 85.

> I can also think of this as 100%. 500% is 5 groups of 100%, so if I have 5 groups of 17, I have 5×17, which is also 85.

   17% of 500 is ___**85**___.

G6-M1-Lesson 27: Solving Percent Problems

1. The Soccer Club of Mathematica County is hosting its annual buffet. 40 players are attending this event. 28 players have either received their food or are currently in line. The rest are patiently waiting to be called to the buffet line. What percent of the players are waiting?

$$\frac{12}{40} = \frac{30}{100} = 30\%$$

30% *of the players are waiting.*

> Since 28 players have either received their food or are in line, I will subtract this from 40 to find out how many players are still waiting. $40 - 28 = 12$. I know 12 is a part of 40 (the whole). $\frac{12}{40} = \frac{3}{10} = \frac{30}{100} = 30\%$

2. Dry Clean USA has finished cleaning 25% of their 724 orders. How many orders do they still need to finish cleaning?

They cleaned 181 *orders, so they still have* 543 *orders left to clean.*

0	181	362	543	724
0	25	50	75	100

> 25% is equal to $\frac{1}{4}$, so $724 \div 4 = 181$. I know 181 orders have been cleaned. Since there are 724 total orders, $724 - 181 = 543$. I know they still have 543 orders to clean.
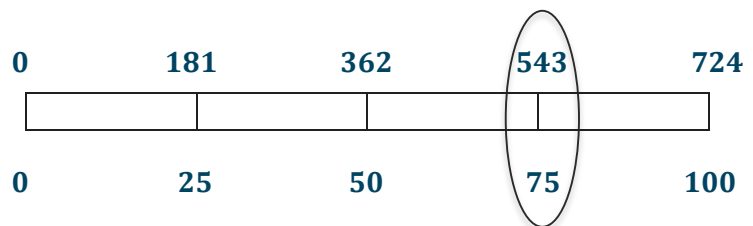
> 25% of the orders have already been cleaned. If 724 is the whole and there are 4 parts, 1 part is equal to 181, and 3 parts are equal to 543.

G6-M1-Lesson 28: Solving Percent Problems

The school fundraiser was a huge success. Ms. Baker's class is in charge of delivering the orders to the students by the end of the day. They delivered 46 orders so far. If this number represents 20% of the total number of orders, how many total orders will Ms. Baker's class have to deliver before the end of the day?

Mrs. Baker's class has to deliver 230 total orders.

$$20\% = \frac{20}{100} = \frac{2}{10} = \frac{46}{230}$$

I know 20% is equal to $\frac{20}{100}$, which can be renamed as $\frac{2}{10}$. I know 46 is a part, and I need to find the whole. I will determine a fraction equivalent to $\frac{2}{10}$, or 20%, by multiplying the numerator and denominator by 23 because $46 \div 2 = 23$. $2 \times 23 = 46$ and $10 \times 23 = 230$.

G6-M1-Lesson 29: Solving Percent Problems

1. Tony completed filling 12 out of a total of 15 party bags for his little sister's party. What percent of the bags does Tony still have to fill?

   20% of the bags still need to be filled.

   Since there are a total of 15 bags and Tony already filled 12, he has 3 bags left to fill. Now I have to find out what percent 3 out of a total of 15 is. I will write a fraction $\frac{3}{15}$ and rename the fraction as $\frac{1}{5}$. I know $\frac{1}{5} = \frac{20}{100}$. So, Tony has to fill 20% of the bags.
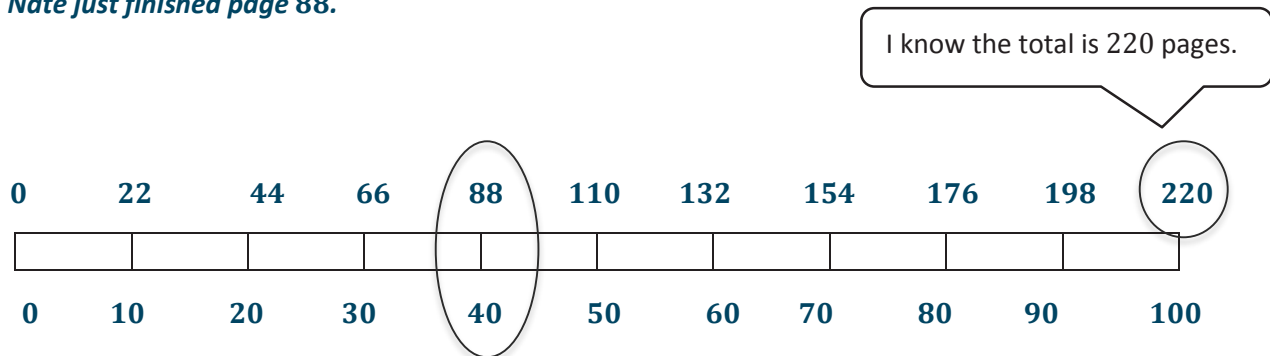
2. Amanda got a 95% on her math test. She answered 19 questions correctly. How many questions were on the test?

   There were 20 questions on the test.

   I know 19 is a part, and we have to find the total. I will write 95% as a fraction and find an equivalent fraction where 19 is the part. $95\% = \frac{95}{100} = \frac{19}{20}$. $95 \div 5 = 19$, and $100 \div 5 = 20$.

3. Nate read 40% of his book containing 220 pages. What page did he just finish?

Nate just finished page 88.

> I know the total is 220 pages.

| 0 | 22 | 44 | 66 | 88 | 110 | 132 | 154 | 176 | 198 | 220 |

| 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |

> I can divide a tape diagram into 10 parts, where each part represents 10%. I know the total is 220, which represents 100%, so I will divide 220 by 10 to determine the value of 10%. 10% is equal to 22. I will use this information to determine equivalent fractions and label the rest of my tape diagram. I know 40% is 88, so Nate just finished page 88.