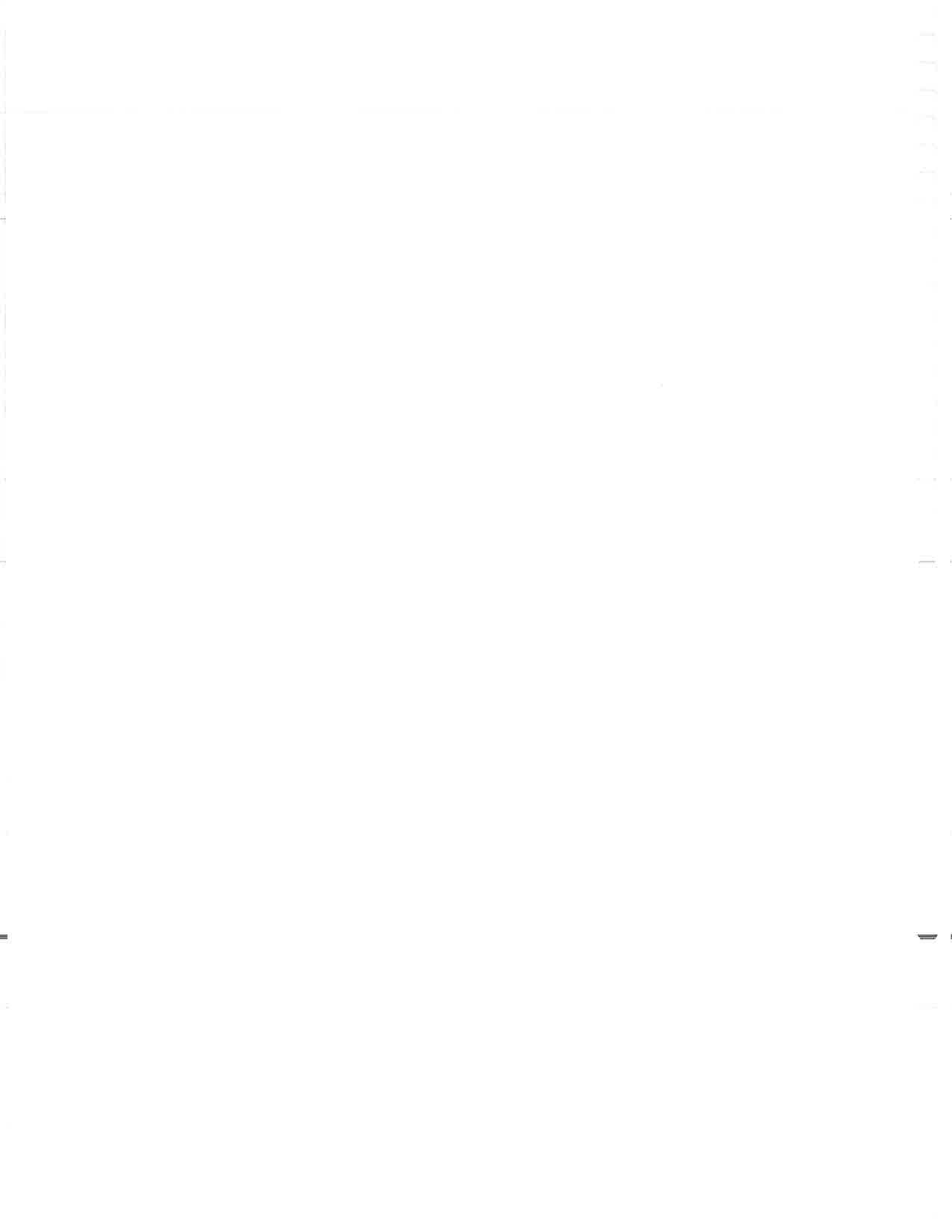


Homework Helpers

Grade 6

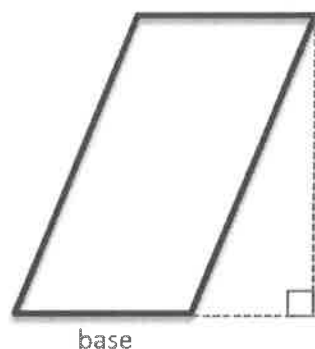
Module 5



G6-M5-Lesson 1: The Area of Parallelograms Through Rectangle

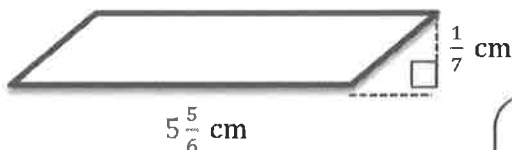
Facts

1. Draw and label the height of the parallelogram below.



I know the height represents a line segment that is perpendicular to the base and whose endpoint is on the opposite side of the parallelogram.

2. Calculate the area of this parallelogram. This figure is not drawn to scale.



I know the base of the figure is $5\frac{5}{6}$ cm and the height of the figure is $\frac{1}{7}$ cm. I can substitute these values into the equation to calculate the area.

$$A = bh$$

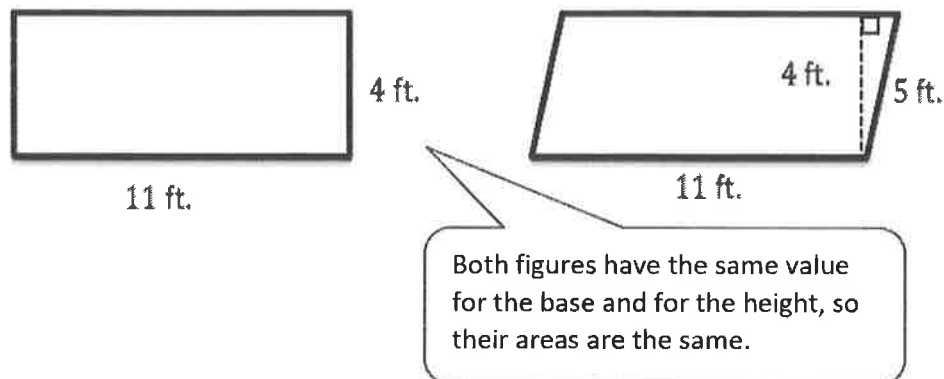
$$= \left(5\frac{5}{6} \text{ cm}\right) \left(\frac{1}{7} \text{ cm}\right)$$

$$= \left(\frac{35}{6} \text{ cm}\right) \left(\frac{1}{7} \text{ cm}\right)$$

$$= \frac{35}{42} \text{ cm}^2$$

In order to efficiently multiply these numbers, I rename $5\frac{5}{6}$ as a fraction greater than one. Since $\frac{6}{6}$ is equal to one, $\frac{30}{6}$ is equal to 5. When I add the remaining $\frac{5}{6}$, the resulting fraction is $\frac{35}{6}$.

3. Do the rectangle and parallelogram below have the same area? Explain why or why not. Note that the figures are not drawn to scale.



Yes, the rectangle and parallelogram have the same area because I can decompose a right triangle on the right side of the parallelogram and move it over to the left side. This transforms the parallelogram into a rectangle. At this time, both rectangles would have the same dimensions; therefore, their areas are the same.

4. A parallelogram has an area of 57.76 square centimeters and a base of 15.2 centimeters. Write an equation that relates the area to the base and height, h . Solve the equation to determine the length of the height.

$$57.76 \text{ cm}^2 = (15.2 \text{ cm})h$$

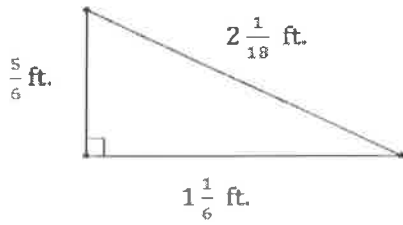
$$57.76 \text{ cm}^2 \div 15.2 \text{ cm} = (15.2 \text{ cm})h \div 15.2 \text{ cm}$$

$$3.8 \text{ cm} = h$$

The height of the parallelogram is 3.8 cm.

G6-M5-Lesson 2: The Area of Right Triangles

1. Calculate the area of the right triangle below. The figure is not drawn to scale.

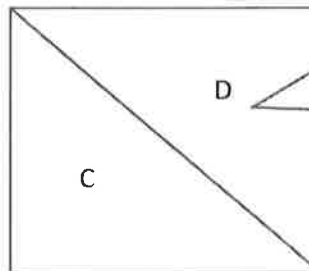
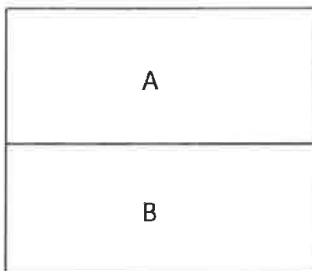


I can substitute the values for base and height into the formula for area of a triangle.

$$A = \frac{1}{2}bh = \frac{1}{2}\left(1\frac{1}{6}\text{ ft.}\right)\left(\frac{5}{6}\text{ ft.}\right) = \frac{1}{2}\left(\frac{7}{6}\text{ ft.}\right)\left(\frac{5}{6}\text{ ft.}\right) = \frac{35}{72}\text{ ft}^2$$

Before I multiply, I need to rename the mixed number as a fraction greater than one.

2. Elise has two rugs at her house. Both rugs have the same length and same width. Elise cut one rug horizontally across the middle, and she made a diagonal cut through the other rug.



If I think about folding two identical pieces of paper to represent these two situations, each new section would represent exactly half of the original piece of paper.

After making the cuts, which rug (labeled A, B, C, or D) has the larger area? Explain.

After making the cuts, the new rugs are all the same size. The horizontal line goes through the center of the rectangle, making two equal parts. The diagonal line also splits the rectangle in two equal parts because the area of a right triangle is exactly half the area of the rectangle.

3. Give the dimensions of a right triangle and a parallelogram with the same area. Explain how you know.

A right triangle has a base of 10 cm and a height of 2 cm.

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}(10 \text{ cm})(2 \text{ cm})$$

$$A = 10 \text{ cm}^2$$

The area of the triangle is 10 cm^2 . In order to find the dimensions of a parallelogram that has the same area, I would need to find factors of 10 because I multiply the base and height of a parallelogram to find the area. Therefore, the parallelogram can have dimensions of 1 cm and 10 cm or 2 cm and 5 cm.

4. If the area of a right triangle is $\frac{7}{20}$ square feet and the height is $\frac{1}{5}$ feet, write an equation that relates the area to the base, b , and the height. Solve the equation to determine the base.

$$\frac{7}{20} \text{ ft}^2 = \frac{1}{2}b \left(\frac{1}{5} \text{ ft.} \right)$$

$$\frac{7}{20} \text{ ft}^2 = \left(\frac{1}{10} \text{ ft.} \right) b$$

$$\frac{7}{20} \text{ ft}^2 \div \frac{1}{10} \text{ ft.} = \left(\frac{1}{10} \text{ ft.} \right) b \div \frac{1}{10} \text{ ft.}$$

$$\frac{7}{2} \text{ ft.} = b$$

$$3 \frac{1}{2} \text{ ft.} = b$$

Since $\frac{7}{2}$ is a fraction greater than one, I can rename this fraction as a mixed number. $\frac{6}{2} = 3$, and 3 plus the remaining $\frac{1}{2}$ is $3 \frac{1}{2}$.

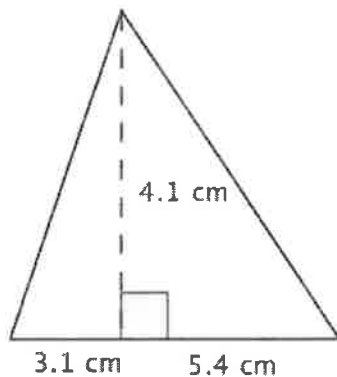
Here I can use the identity $a \cdot b \div b = a$ and divide both sides of the equation by $\frac{1}{10}$.

Therefore, the base of the right triangle is $3 \frac{1}{2}$ ft.

G6-M5-Lesson 3: The Area of Acute Triangles Using Height and Base

Calculate the area of each shape below. Figures are not drawn to scale.

1.



$$A = \frac{1}{2}(3.1 \text{ cm})(4.1 \text{ cm}) = 6.355 \text{ cm}^2$$

$$A = \frac{1}{2}(5.4 \text{ cm})(4.1 \text{ cm}) = 11.07 \text{ cm}^2$$

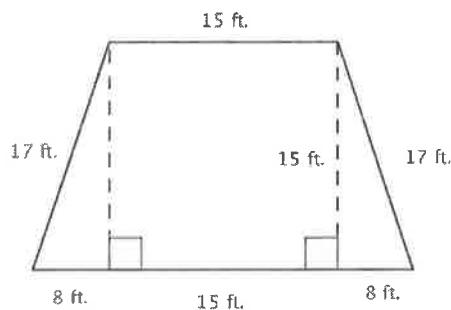
$$A = 6.355 \text{ cm}^2 + 11.07 \text{ cm}^2 = 17.425 \text{ cm}^2$$

OR

$$A = \frac{1}{2}(8.5 \text{ cm})(4.1 \text{ cm}) = 17.425 \text{ cm}^2$$

I can decompose the large triangle into two smaller triangles and find the area of each smaller triangle using the formula $A = \frac{1}{2}bh$ and then add the two areas. The sum of both areas is the total area of the triangle. Or, I can find the length of the base by adding 3.1 cm and 5.4 cm and then use the formula to determine the total area.

2.



I can decompose the trapezoid into two right triangles and a square. I can calculate the area of each of these smaller shapes and then find the sum of these areas, which is the total area of the figure.

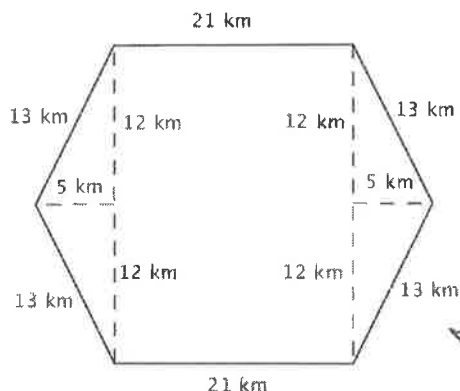
$$A = \frac{1}{2}(8 \text{ ft.})(15 \text{ ft.}) = 60 \text{ ft}^2$$

$$A = (15 \text{ ft.})(15 \text{ ft.}) = 225 \text{ ft}^2$$

$$A = \frac{1}{2}(8 \text{ ft.})(15 \text{ ft.}) = 60 \text{ ft}^2$$

$$A = 60 \text{ ft}^2 + 225 \text{ ft}^2 + 60 \text{ ft}^2 = 345 \text{ ft}^2$$

3.



$$A = \frac{1}{2}(24 \text{ km})(5 \text{ km}) = 60 \text{ km}^2$$

$$A = (21 \text{ km})(24 \text{ km}) = 504 \text{ km}^2$$

$$A = \frac{1}{2}(24 \text{ km})(5 \text{ km}) = 60 \text{ km}^2$$

$$A = 60 \text{ km}^2 + 504 \text{ km}^2 + 60 \text{ km}^2 = 624 \text{ km}^2$$

I can decompose this hexagon into three parts. There is a triangle on each side and a rectangle in the middle. I can calculate the area of the entire figure by finding the sum of the areas of the smaller shapes.

4. Jasmine is building an enclosure for her rabbits. The bottom of the enclosure is in the shape of a triangle, with a base of 72 inches and an altitude of 36 inches. How much space will her rabbits have?

$$A = \frac{1}{2}bh = \frac{1}{2}(72 \text{ in.})(36 \text{ in.}) = 1,296 \text{ in}^2$$

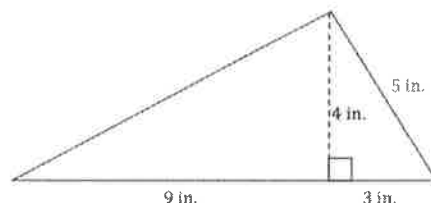
Jasmine's rabbits will have 1,296 in² in which to play.

By calculating the area of the space, I can determine how much room there is inside the enclosure, which is where the rabbits will play.

5. Examine the triangle to the right.

- a. Write an expression to show how you would calculate the area.

$$\frac{1}{2}(9 \text{ in.})(4 \text{ in.}) + \frac{1}{2}(3 \text{ in.})(4 \text{ in.}) \text{ or } \frac{1}{2}(12 \text{ in.})(4 \text{ in.})$$



- b. Identify each part of your expression as it relates to the triangle.

In the first expression, I decomposed the larger triangle into two smaller triangles. The base is composed of 9 in. and 3 in., and the height, or the altitude, is 4 in. The sum of the areas of these two triangles is the total area of the triangle.

In the second expression, the base of the larger triangle is 12 in. because 9 in. + 3 in. = 12 in., and the height, or the altitude, is 4 in.

6. A room has a triangular floor with an area of $16\frac{1}{4}$ sq. m. If the altitude of the triangle is $3\frac{1}{2}$ m, write an equation to determine the length of the base, b , in meters. Then solve the equation.

To solve for b , I can use the identity $a \cdot b \div b = a$.

$$16\frac{1}{4} \text{ m}^2 = \frac{1}{2} b \left(3\frac{1}{2} \text{ m} \right)$$

$$\frac{65}{4} \text{ m}^2 = \left(\frac{7}{4} \text{ m} \right) b$$

$$\frac{65}{4} \text{ m}^2 \div \frac{7}{4} \text{ m} = \left(\frac{7}{4} \text{ m} \right) b \div \frac{7}{4} \text{ m}$$

$$\frac{65}{7} \text{ m}^2 = b$$

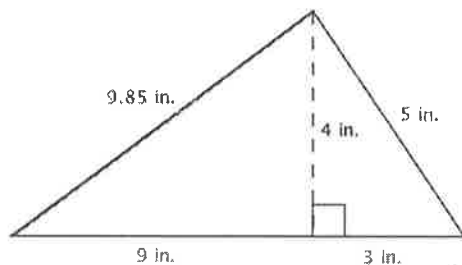
$$9\frac{2}{7} \text{ m}^2 = b$$

I can substitute the given values for area and height into the formula and rename the mixed numbers as fractions greater than one to efficiently multiply.

After dividing the fractions greater than one, I can rename the fraction greater than one, $\frac{65}{7}$, as a mixed number.

G6-M5-Lesson 4: The Area of All Triangles Using Height and Base

1. Calculate the area of this triangle. This figure is not drawn to scale.

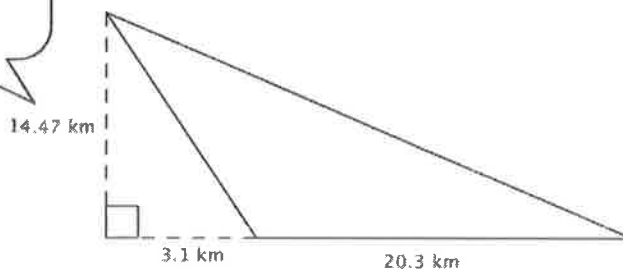


$$A = \frac{1}{2}(12 \text{ in.})(4 \text{ in.}) = 24 \text{ in}^2$$

To determine the area of the triangle, I need to know the base and height. The base of the triangle is the sum of 9 in. and 3 in., which is 12 in. The height is 4 in.

2. Calculate the area of this triangle. This figure is not drawn to scale.

The height of the obtuse triangle is found outside of the triangle.



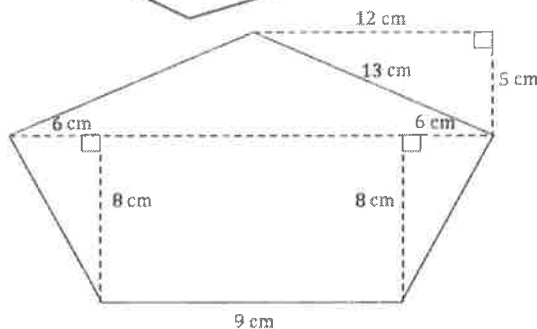
$$A = \frac{1}{2}(20.3 \text{ km})(14.47 \text{ km}) = 146.8705 \text{ km}^2$$

When I determine the length of the base, I remember not to use the length between the base and the altitude.

I need to decompose the given figure in order to calculate the total area.

3. Calculate the area of the figure below. This figure is not drawn to scale.

I can determine the total length of the base of the triangle in the upper region of this figure by adding 6 cm + 9 cm + 6 cm.



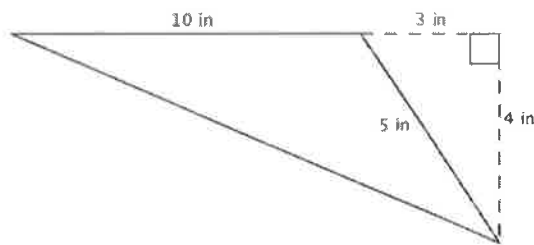
$$A = \frac{1}{2}(6 \text{ cm})(8 \text{ cm}) = 24 \text{ cm}^2$$

$$A = \frac{1}{2}(21 \text{ cm})(5 \text{ cm}) = 52.5 \text{ cm}^2$$

$$A = (9 \text{ cm})(8 \text{ cm}) = 72 \text{ cm}^2$$

$$A = 24 \text{ cm}^2 + 24 \text{ cm}^2 + 52.5 \text{ cm}^2 + 72 \text{ cm}^2 = 172.5 \text{ cm}^2$$

4. Jason and Joelle are both trying to calculate the area of an obtuse triangle. Examine their calculations below.

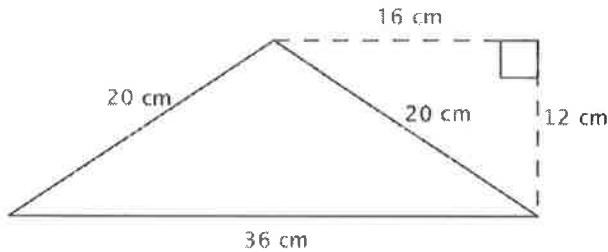


| Jason's Work | Joelle's Work |
|--|---|
| $A = \frac{1}{2} \times 10 \text{ in.} \times 4 \text{ in.}$ | $A = \frac{1}{2} \times 3 \text{ in.} \times 4 \text{ in.}$ |
| $A = 20 \text{ in}^2$ | $A = 6 \text{ in}^2$ |

Which student calculated the area correctly? Explain why the other student is not correct.

Jason calculated the area correctly. Although Joelle did use the altitude of the triangle, she used the length between the altitude and the base rather than the length of the actual base.

5. David calculated the area of the triangle below. His work is shown.



David correctly determined the base and height and substituted these values into the formula to determine the area of this triangle.

$$A = \frac{1}{2} \times 36 \text{ cm} \times 12 \text{ cm}$$

$$A = 216 \text{ cm}^2$$

Although David was told his work is correct, he had a hard time explaining why it is correct. Help David explain why his calculations are correct.

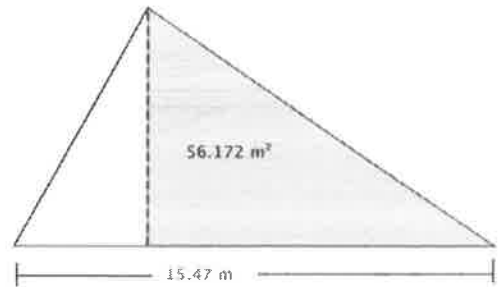
The formula for the area of a triangle is $A = \frac{1}{2}bh$. David followed this formula because 12 cm is the height of the triangle, and 36 cm is the base of the triangle.

6. The larger triangle below has a base of 15.47 m; the gray triangle has an area of 56.172 m^2 .

- a. Determine the area of the larger triangle if it has a height of 11.3 m.

$$A = \frac{1}{2}(15.47 \text{ m})(11.3 \text{ m})$$

$$A = 87.4055 \text{ m}^2$$



- b. Let A be the area of the unshaded (white) triangle in square meters. Write and solve an equation to determine the value of A using the areas of the larger triangle and the gray triangle.

$$56.172 \text{ m}^2 + A = 87.4055 \text{ m}^2$$

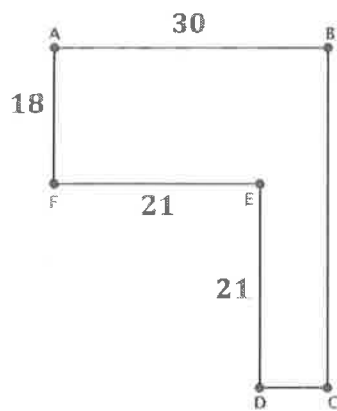
$$56.172 \text{ m}^2 + A - 56.172 \text{ m}^2 = 87.4055 \text{ m}^2 - 56.172 \text{ m}^2$$

$$A = 31.2335 \text{ m}^2$$

The sum of the area of the white triangle and the area of the gray triangle is equal to the area of the larger triangle, 87.4055 m^2 . I know the area of the gray triangle is 56.172 m^2 .

G6-M5-Lesson 5: The Area of Polygons Through Composition and Decomposition

1. If $AB = 30$ units, $FE = 21$ units, $AF = 18$ units, and $DE = 21$ units, find the length of both other sides. Then, find the area of the irregular polygon. All measurements are in units.



I can label the lengths of the sides given in the problem.

To determine the length of \overline{BC} , I can add the lengths AF and DE since their sum is equal to BC .

I know $FE + CD = 30$ units, and I know $FE = 21$ units, so I can write the equation $21 \text{ units} + CD = 30 \text{ units}$ and solve for CD .

$$CD = 30 \text{ units} - 21 \text{ units} = 9 \text{ units}$$

$$BC = 18 \text{ units} + 21 \text{ units} = 39 \text{ units}$$

To calculate the area of the figure, decompose the figure into two rectangles. The dimensions of one rectangle are $30 \text{ units} \times 18 \text{ units}$, and the dimensions of the other rectangle are $9 \text{ units} \times 21 \text{ units}$.

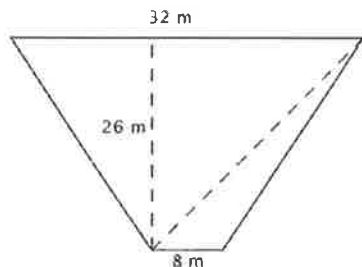
$$A = 30 \text{ units} \times 18 \text{ units} = 540 \text{ units}^2$$

$$A = 9 \text{ units} \times 21 \text{ units} = 189 \text{ units}^2$$

$$\text{Total Area} = 540 \text{ units}^2 + 189 \text{ units}^2 = 729 \text{ units}^2$$

I can decompose this trapezoid into two triangles.

2. Determine the area of the trapezoid below. The trapezoid is not drawn to scale.



Area of Triangle 1

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2} \times 32 \text{ m} \times 26 \text{ m}$$

$$A = 416 \text{ m}^2$$

Area of Triangle 2

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2} \times 8 \text{ m} \times 26 \text{ m}$$

$$A = 104 \text{ m}^2$$

Area of Trapezoid = Area of Triangle 1 + Area of Triangle 2

$$\text{Area of Trapezoid} = 416 \text{ m}^2 + 104 \text{ m}^2 = 520 \text{ m}^2$$

I construct a rectangle around this isosceles trapezoid. To determine the base of each triangle, I subtract 4 m from 21 m and then divide the difference by 2 since the triangles have the same length for the base.

3. Determine the area of the isosceles trapezoid below. The image is not drawn to scale.

Area of Rectangle

$$A = bh$$

$$A = 21 \text{ m} \times 16 \text{ m}$$

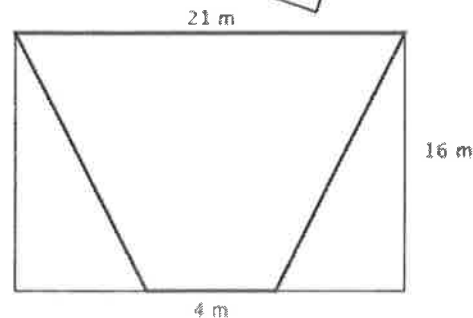
$$A = 336 \text{ m}^2$$

Area of Triangles 1 and 2

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2} \times 8.5 \text{ m} \times 16 \text{ m}$$

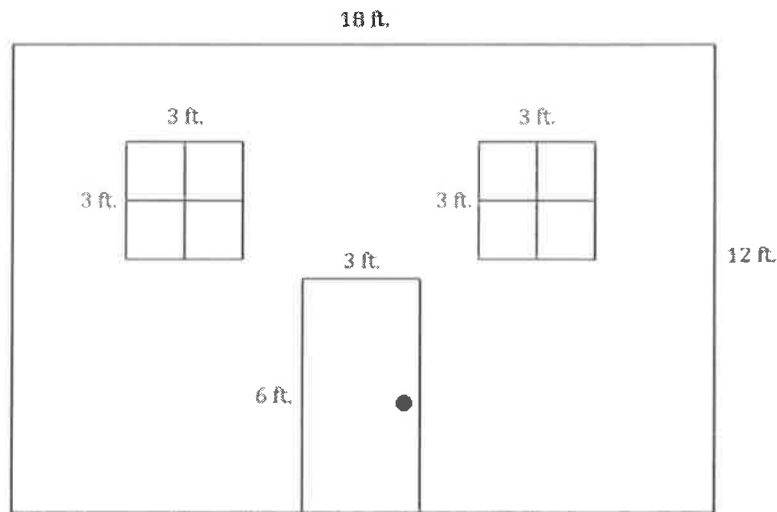
$$A = 68 \text{ m}^2$$



Area of Trapezoid = Area of Rectangle – Area of Triangle 1 – Area of Triangle 2

$$\text{Area of Trapezoid} = 336 \text{ m}^2 - 68 \text{ m}^2 - 68 \text{ m}^2 = 200 \text{ m}^2$$

4. Here is a sketch of a wall that needs to be painted.



- a. The windows and door will not be painted. Calculate the area of the wall that will be painted.

Area of the entire wall: $18 \text{ ft.} \times 12 \text{ ft.} = 216 \text{ ft}^2$

Area of the two windows: $2(3 \text{ ft.} \times 3 \text{ ft.}) = 18 \text{ ft}^2$

Area of the door: $6 \text{ ft.} \times 3 \text{ ft.} = 18 \text{ ft}^2$

Area that needs to be painted: $216 \text{ ft}^2 - 18 \text{ ft}^2 - 18 \text{ ft}^2 = 180 \text{ ft}^2$

The area of the wall that will be painted is 180 ft^2 .

To find the area of the wall that will be painted, I can determine the area of each part that will not be painted (the windows and the door) and subtract the areas of these parts from the area of the entire wall.

- b. If a quart of Cover-All Paint covers 45 ft^2 , how many quarts must be purchased for the painting job?

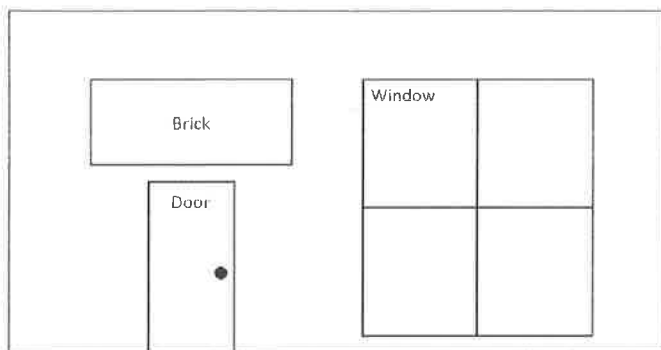
$$180 \div 45 = 4$$

Therefore, 4 quarts of paint must be purchased.

Since 180 ft^2 is the total area that needs to be painted, I can divide this amount by the area covered by one quart.

G6-M5-Lesson 6: Area in the Real World

1. Below is a drawing of a wall that will be covered in either wallpaper or paint. The wall is 12 ft. high and 23 ft. long. The window, brick, and door will not be painted. The brick measures 7 ft. \times 36 in., the door is 3 ft. wide and 6 ft. high, and the window is 8 ft. \times 9 ft.



I can label the dimensions of the wall, window, brick, and door. I notice the width of the brick is written in inches (36 in.), so I can convert that measurement to 3 ft. so all the units are feet.

- a. How many square feet of wallpaper are needed to cover the wall?

Area of entire wall: $23 \text{ ft.} \times 12 \text{ ft.} = 276 \text{ ft}^2$

Area of the brick: $7 \text{ ft.} \times 3 \text{ ft.} = 21 \text{ ft}^2$

Area of the door: $3 \text{ ft.} \times 6 \text{ ft.} = 18 \text{ ft}^2$

Area of the window: $8 \text{ ft.} \times 9 \text{ ft.} = 72 \text{ ft}^2$

Area that will be painted: $276 \text{ ft}^2 - (18 \text{ ft}^2 + 21 \text{ ft}^2 + 72 \text{ ft}^2) = 165 \text{ ft}^2$

I can find the area of each part of the wall that is not going to be covered and add the areas together. Then I can subtract that total from the total area of the wall.

- b. The wallpaper is sold in rolls that are 18 in. wide and 30 ft. long. Rolls of solid color wallpaper will be used, so patterns do not have to match up.

- i. What is the area of one roll of wallpaper?

Area of one roll of wallpaper: $1.5 \text{ ft.} \times 30 \text{ ft.} = 45 \text{ ft}^2$

I have to pay close attention to the units.

- ii. How many rolls would be needed to cover the wall?

$165 \text{ ft}^2 \div 45 \text{ ft}^2 \approx 3.7$

It is necessary to buy 4 rolls of wallpaper.

I have to round up because I can't purchase part of a roll of wallpaper.

- c. This week, the rolls of wallpaper are on sale for \$9.99 per roll. Find the cost of covering the wall with wallpaper.

$$\$9.99 \times 4 = \$39.96$$

We need four rolls of wallpaper to cover the wall, which will cost \$39.96.

- d. A gallon of special textured paint covers 200 ft^2 and is on sale for \$19.99 per gallon. The wall needs to be painted twice (the wall needs two coats of paint). Find the cost of using paint to cover the wall.

$$165 \text{ ft}^2 \times 2 = 330 \text{ ft}^2$$

If the wall needs to be painted twice, we need to paint a total area of 330 ft^2 .

$$330 \text{ ft}^2 \div 200 \text{ ft}^2 = 1.65$$

Two gallons of paint need to be purchased in order to paint the wall with two coats of paint.

$$\$19.99 \times 2 = \$39.98$$

The cost for two coats of paint is \$39.98.

2. A classroom has a length of 27 ft. and a width of 32 ft. The flooring is to be replaced by tiles. If each tile has a length of 18 in. and a width of 36 in., how many tiles are needed to cover the classroom floor?

Area of the classroom: $27 \text{ ft.} \times 32 \text{ ft.} = 864 \text{ ft}^2$

Area of each tile: $1.5 \text{ ft.} \times 3 \text{ ft.} = 4.5 \text{ ft}^2$

$$\frac{\text{Area of the classroom}}{\text{Area of each tile}} = \frac{864 \text{ ft}^2}{4.5 \text{ ft}^2} = 192$$

192 tiles are needed to cover the classroom floor.

To find the area of one tile, I can convert the dimensions from inches to feet and then multiply.

3. Challenge: Assume that the tiles from Problem 2 are unavailable. Another design is available, but the tiles are square, 18 in. on a side. If these are to be installed, how many must be ordered?

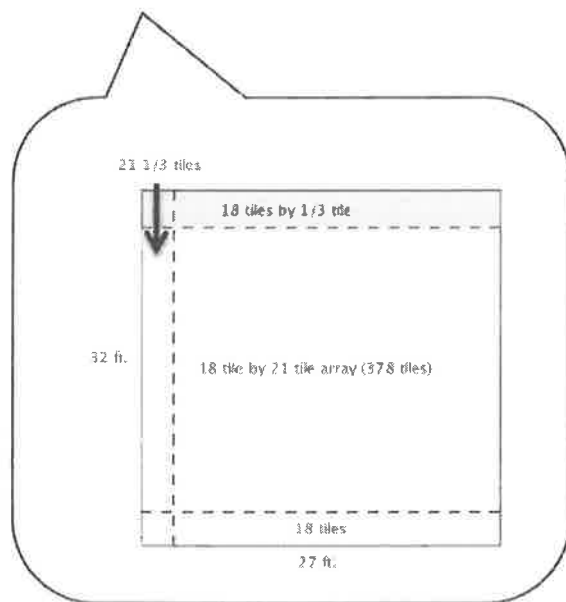
Solutions will vary. An even number of tiles fit on the 27 foot length of the room (18 tiles), but the width requires $21\frac{1}{3}$ tiles.

I know the length of the room is 27 ft., so I can divide 27 by 1.5 since the length of one 18 in. tile is 1.5 ft. Since I know the width of the room is 32 ft., I can divide 32 by 1.5.

This accounts for an 18 tile by 21 tile array. $18 \times 21 = 378$, so 378 tiles must be ordered.

The remaining area is 27 ft. \times 0.5 ft. (18 tiles \times $\frac{1}{3}$ tile)

The remaining area is the part in the upper portion of the image to the right. The length is 27 ft. (18 tiles), and the width is $\frac{1}{3}$ tile. Since the tile is 1.5 ft. long, $\frac{1}{3}$ of 1.5 ft. is 0.5 ft.



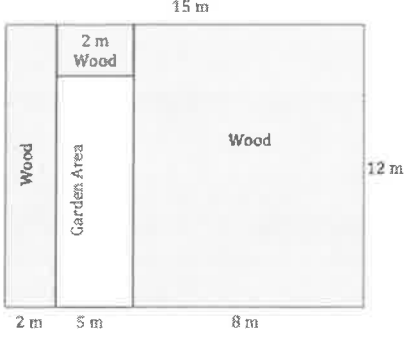
Since 18 of the $\frac{1}{3}$ tiles are needed, 6 additional tiles must be cut because $\frac{18}{3} = 6$.

Using the same logic as above, some students may correctly say they will need 384 tiles.

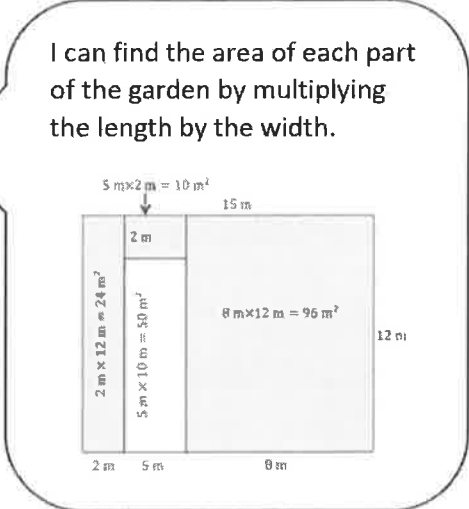
I can add the number of tiles in the 18 tile by 21 tile array, 378 tiles, with the number of tiles needed for the remaining part, 6 tiles. $378 \text{ tiles} + 6 \text{ tiles} = 384 \text{ tiles}$.

4. Henry's deck has an open area where he would like to plant a vegetable garden.
- a. Find the missing portion of the deck. Write the expression and evaluate it.

The width of the missing portion is 5 m, and the length is $12\text{ m} - 2\text{ m}$, which is 10 m.



I can find the area of each part of the garden by multiplying the length by the width.



$$5\text{ m} \times 10\text{ m} = 50\text{ m}^2$$

OR

$$15\text{ m} \times 12\text{ m} - 2\text{ m} \times 12\text{ m} - 5\text{ m} \times 2\text{ m} - 8\text{ m} \times 12\text{ m} = 50\text{ m}^2$$

- b. Find the missing portion of the deck using a different method. Write the expression and evaluate it.
Choose whichever method was not used in part (a).
- c. Write your two equivalent expressions.

$$5\text{ m} \times 10\text{ m}$$

$$15\text{ m} \times 12\text{ m} - 2\text{ m} \times 12\text{ m} - 5\text{ m} \times 2\text{ m} - 8\text{ m} \times 12\text{ m}$$

- d. Explain how each demonstrates a different understanding of the diagram.

One expression shows the dimensions of the garden area (interior rectangle, $5\text{ m} \times 10\text{ m}$), and one shows finding the total area minus each of the three wooden areas.

G6-M5-Lesson 7: Distance on the Coordinate Plane

1. Given the pairs of points, determine whether the segment that joins them will be horizontal, vertical, or neither.

a. $X(6, 1)$ and $Y(-4.5, 1)$

Horizontal

The y -coordinates are the same, so a horizontal segment can connect these points.

b. $M(2, -3)$ and $N(-2, 3)$

Neither

The x - and y -coordinates are different, so neither a horizontal nor a vertical line can connect these points.

c. $E(-12, 5)$ and $F(-12, 8)$

Vertical

The x -coordinates are the same, so a vertical segment can connect these points.

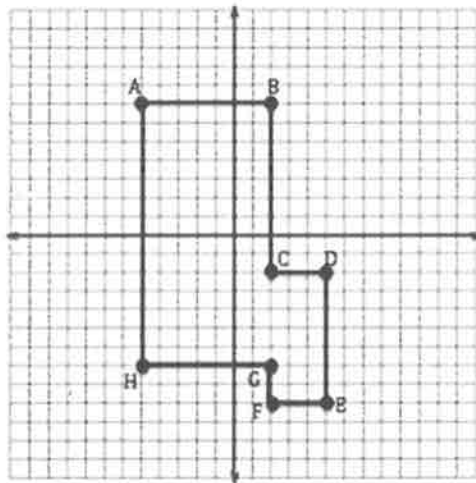
2. Complete the table using absolute value to determine the lengths of the line segments.

I will focus on the coordinates that are different. Here, the x -coordinates are different. They have different signs, so they are on opposite sides of 0. I will add the absolute value of each coordinate to find the distance, or the length of the line segment that will connect the points.

| Line Segment | Point | Point | Distance | Proof |
|-----------------|-----------|-----------|----------|-------------------|
| \overline{AB} | $(-2, 4)$ | $(5, 4)$ | 7 | $ -2 + 5 = 7$ |
| \overline{CD} | $(2, -6)$ | $(2, -3)$ | 3 | $ -6 - -3 = 3$ |

Here, the y -coordinates are different, and they have the same sign, so they are on the same side of 0. I will subtract the absolute value of each coordinate to find the distance, or the length of the line segment that will connect the points.

3. Complete the table using the diagram and absolute value to determine the lengths of the line segments.



| Line Segment | Point | Point | Distance | Proof |
|-----------------|------------|------------|----------|-------------------|
| \overline{AB} | $(-5, 7)$ | $(2, 7)$ | 7 | $ -5 + 2 = 7$ |
| \overline{BC} | $(2, 7)$ | $(2, -2)$ | 9 | $ 7 + -2 = 9$ |
| \overline{CD} | $(2, -2)$ | $(5, -2)$ | 3 | $ 5 - 2 = 3$ |
| \overline{DE} | $(5, -2)$ | $(5, -9)$ | 7 | $ -9 - -2 = 7$ |
| \overline{EF} | $(5, -9)$ | $(2, -9)$ | 3 | $ 5 - 2 = 3$ |
| \overline{FG} | $(2, -9)$ | $(2, -7)$ | 2 | $ -9 - -7 = 2$ |
| \overline{GH} | $(2, -7)$ | $(-5, -7)$ | 7 | $ 2 + -5 = 7$ |
| \overline{HA} | $(-5, -7)$ | $(-5, 7)$ | 14 | $ -7 + 7 = 14$ |

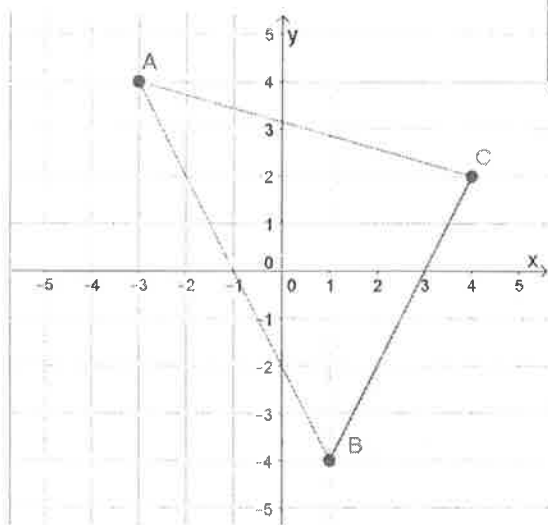
I can use absolute value just like I did in Problem 2 to determine the lengths of the line segments. I can also use the diagram in this problem to count the distance between each point to check my answer.

G6-M5-Lesson 8: Drawing Polygons in the Coordinate Plane

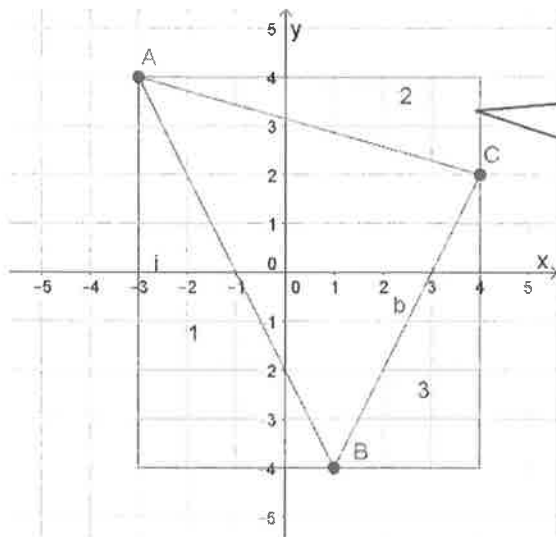
Plot the points for each shape, determine the area of the polygon, and then write an expression that could be used to determine the area of the figure. Explain how each part of the expression corresponds to the situation.

1. $A(-3, 4)$, $B(1, -4)$, $C(4, 2)$

The shape formed is a triangle.



Points are written as (x, y) . So the first number tells me to move left or right from the origin, and the second number tells me to move up or down from the origin.



I am unable to determine the base and height of the triangle by counting. Instead, I can draw a rectangle around the triangle.

Area of Rectangle

$$A = lw$$

$$A = (7 \text{ units})(8 \text{ units})$$

$$A = 56 \text{ units}^2$$

Area of Triangle 1

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}(8 \text{ units})(4 \text{ units})$$

$$A = 16 \text{ units}^2$$

Area of Triangle 2

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}(7 \text{ units})(2 \text{ units})$$

$$A = 7 \text{ units}^2$$

Area of Triangle 3

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}(6 \text{ units})(3 \text{ units})$$

$$A = 9 \text{ units}^2$$

Total Area of Triangle

$$A = 56 \text{ units}^2 - 16 \text{ units}^2 - 7 \text{ units}^2 - 9 \text{ units}^2$$

$$A = 24 \text{ units}^2$$

I can subtract the area of each outer triangle from the area of the rectangle, and I will be left with the area of the given triangle.

The area of the triangle is 24 units^2 .

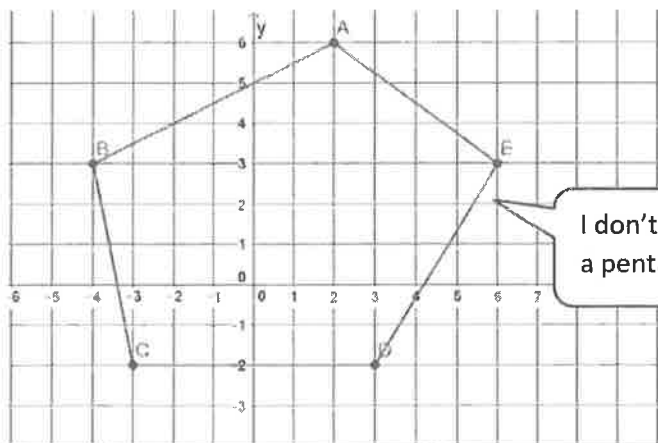
Expression: $(7)(8) - \frac{1}{2}(8)(4) - \frac{1}{2}(7)(2) - \frac{1}{2}(6)(3)$

The first term in the expression is the area of a rectangle that goes around the given triangle.

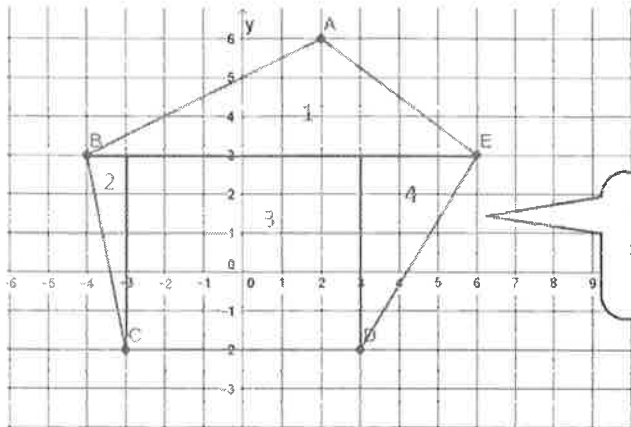
The other terms represent the area of each triangle that surrounds the given triangle.

2. $A(2, 6), B(-4, 3), C(-3, -2), D(3, -2), E(6, 3)$

The shape formed is a pentagon.



I don't know a formula for calculating the area of a pentagon. I will need to use another method.



I try to decompose the shape into several smaller polygons instead of drawing the rectangle around the outside.

Area of Triangle 1

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}(10 \text{ units})(3 \text{ units})$$

$$A = 15 \text{ units}^2$$

Area of Triangle 2

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}(1 \text{ unit})(5 \text{ units})$$

$$A = 2.5 \text{ units}^2$$

Area of Rectangle 3

$$A = lw$$

$$A = (6 \text{ units})(5 \text{ units})$$

$$A = 30 \text{ units}^2$$

Area of Triangle 4

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}(3 \text{ units})(5 \text{ units})$$

$$A = 7.5 \text{ units}^2$$

Total Area of Pentagon

$$A = 15 \text{ units}^2 + 2.5 \text{ units}^2 + 30 \text{ units}^2 + 7.5 \text{ units}^2$$

$$A = 55 \text{ units}^2$$

Now that I have the area of all the parts, I can add them together to get the area of the pentagon.

The area of the pentagon is 55 units².

Expression: $(6)(5) + \frac{1}{2}(10)(3) + \frac{1}{2}(1)(5) + \frac{1}{2}(3)(5)$

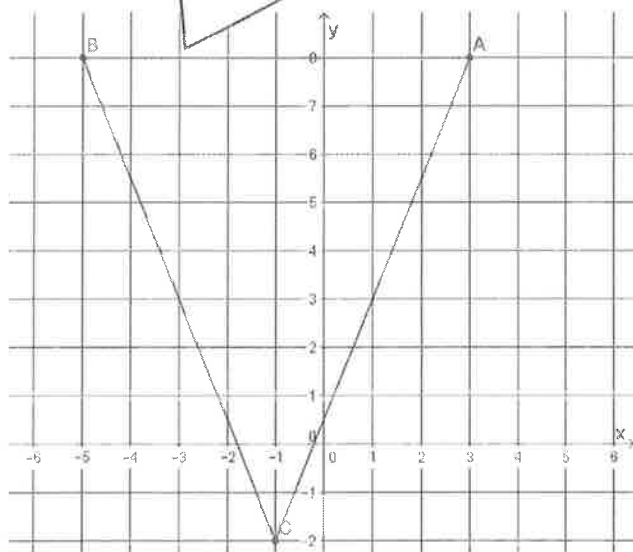
The first term in the expression is the area of a rectangle that is part of the pentagon. Each of the other terms represents the area of each of the triangles that make up the rest of the pentagon.

3. A triangle with vertices located at $(-5, 8)$ and $(3, 8)$ has an area of 40 square units. Determine one possible location of the other vertex.

I know that the area of a triangle is half the area of a rectangle with the same base and height. 40 is half of 80, so the product of the base and height must be equal to 80.

One possible location of the third point would be $(-1, -2)$.

The two given points are 8 units apart. This could be the base. Now I need a third point that is 10 units from the base so that the height will be equal to 10 units.



The area of this triangle would be

$$A = \frac{1}{2}bh$$

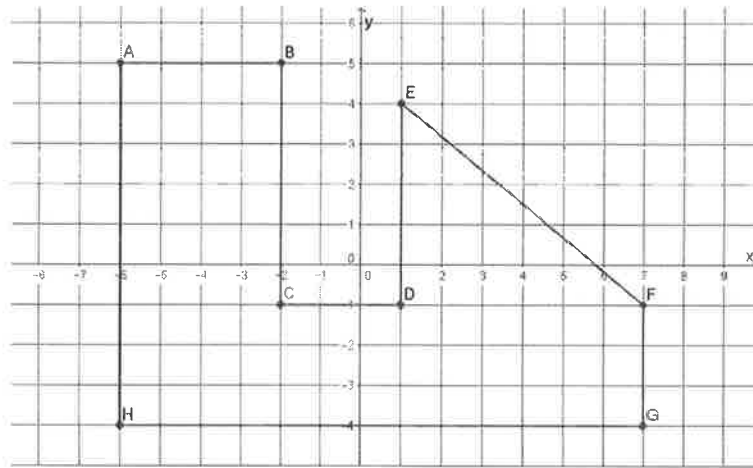
$$A = \frac{1}{2}(8 \text{ units})(10 \text{ units})$$

$$A = 40 \text{ units}^2$$

I could have chosen any point on the line $y = -2$ to form the triangle. I could have also gone up ten units from the base instead of down to form the third point.

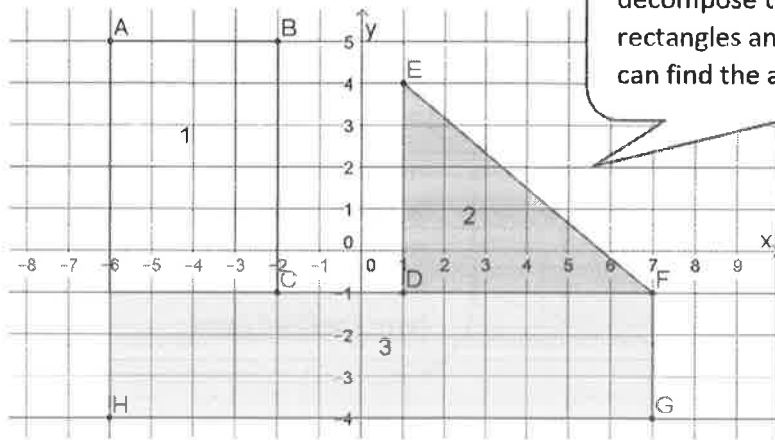
G6-M5-Lesson 9: Determining Area and Perimeter of Polygons on the Coordinate Plane

Use the diagram to answer the following questions.



- a. Determine the area of the polygon.

Similar to Lesson 8, I can decompose the polygon into rectangles and triangles. Then I can find the area of each piece.



Area of Rectangle 1

$$A = lw$$

$$A = (4 \text{ units})(6 \text{ units})$$

$$A = 24 \text{ units}^2$$

Area of Triangle 2

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}(6 \text{ units})(5 \text{ units})$$

$$A = 15 \text{ units}^2$$

Area of Rectangle 3

$$A = lw$$

$$A = (13 \text{ units})(3 \text{ units})$$

$$A = 39 \text{ units}^2$$

Total Area of Polygon

$$A = 24 \text{ units}^2 + 15 \text{ units}^2 + 39 \text{ units}^2$$

$$A = 78 \text{ units}^2$$

- b. Write an expression that could be used to determine the area.

$$(4)(6) + \frac{1}{2}(6)(5) + (13)(3)$$

I write the expression by showing how I determine the area of each piece and then add them together.

- c. Describe another method you could use to find the area of the polygon. Then, state how the expression for the area would be different from the expression you wrote in part (b).

Instead, I could have drawn a rectangle around the outside of the polygon and subtracted the area of the extra pieces from the total area.

- d. If the length of each square was worth 5 units instead of 1 unit, how would the area of polygon change? How would your expression change to represent this area?

$$5l \times 5w = 25lw$$

If each square is 5 units instead of 1 unit, then the length would be 5 times as long as the original length, and the width would be 5 times as long as the original width.

The area will be 25 times larger than the original area when the side lengths are five times longer than the original lengths. Therefore, I could multiply my entire expression by 25 to make the area of the original polygon 25 times bigger.

$$25 \left[(4)(6) + \frac{1}{2}(6)(5) + (13)(3) \right]$$

G6-M5-Lesson 10: Distance, Perimeter, and Area in the Real World

1. Lukas and Juan are modeling rectangular designs using toothpicks. They have been given 30 toothpicks. They agree that they should only use whole toothpicks and not break any apart.

I could draw pictures of possible rectangles might help get me started.

- a. What are all of the possible dimensions of the rectangular designs?

| <i>Length (in toothpicks)</i> | <i>Width (in toothpicks)</i> |
|-----------------------------------|----------------------------------|
| 14 | 1 |
| 13 | 2 |
| 12 | 3 |
| 11 | 4 |
| 10 | 5 |
| 9 | 6 |
| 8 | 7 |

There are two lengths and two widths in each rectangle. So I want half of the 30 toothpicks, 15, used in one length and one width.

- b. Which rectangular design yields a maximum area? Which design yields the minimum area?

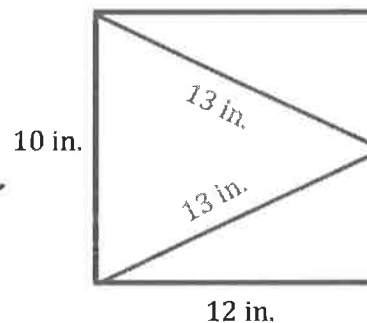
| Length (in toothpicks) | Width (in toothpicks) | Area (in toothpicks squared) |
|---------------------------|--------------------------|---------------------------------|
| 14 | 1 | 14 |
| 13 | 2 | 26 |
| 12 | 3 | 36 |
| 11 | 4 | 44 |
| 10 | 5 | 50 |
| 9 | 6 | 54 |
| 8 | 7 | 56 |

I can create a table of all the possible areas so that I can compare them to answer the question.

The 8 toothpicks by 7 toothpicks design would have the maximum area of 56 toothpicks squared, while the 14 toothpicks by 1 toothpick design would only have an area of 14 toothpicks squared.

2. Maria is designing a new rectangular block for a quilt.

I need to include all of the lines where Maria would be sewing, not just the outside edges. I might want to label some of the sides where the measurements are not shown.



- a. Maria must sew along all of the edges. Determine the total length that must be sewn.

$$10 \text{ in.} + 10 \text{ in.} + 13 \text{ in.} + 13 \text{ in.} + 12 \text{ in.} + 12 \text{ in.} = 70 \text{ in.}$$

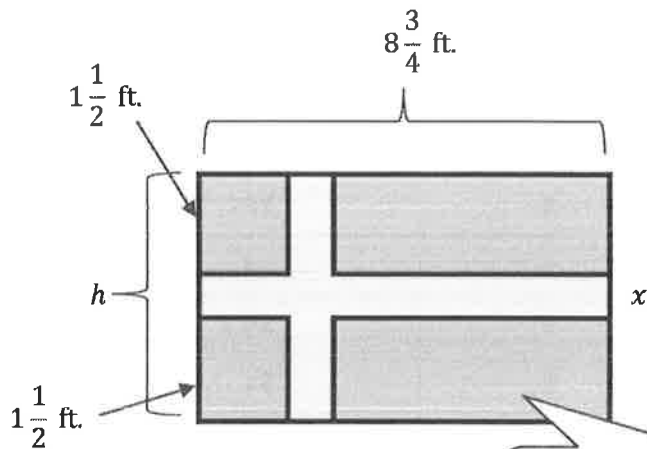
Maria must sew across 70 inches.

- b. If Maria's spool contains 110 yards (3,960 inches) of thread, how many complete blocks could she sew?

$$3,960 \text{ in.} \div 70 \text{ in. per block} \approx 56.57 \text{ blocks}$$

Maria would be able to complete 56 blocks.

3. This is a drawing of the country of Sweden's flag. The area of the flag is $30\frac{5}{8}$ ft².



I have been given the area of the rectangle. So I can work backward to determine the height.

- a. Determine the value of the height of the flag.

$$A = bh$$

$$h = A \div b$$

$$h = 30\frac{5}{8} \text{ ft}^2 \div 8\frac{3}{4} \text{ ft.}$$

$$h = \frac{245}{8} \text{ ft}^2 \div \frac{35}{4} \text{ ft.}$$

$$h = \frac{7}{2} \text{ ft.}$$

$$h = 3\frac{1}{2} \text{ ft.}$$

The height of the flag is $3\frac{1}{2}$ feet.

- b. Using what you found in part (a), determine the missing value of the height.

$$3\frac{1}{2} \text{ ft.} = 1\frac{1}{2} \text{ ft.} + 1\frac{1}{2} \text{ ft.} + x$$

$$3\frac{1}{2} \text{ ft.} = 3 \text{ ft.} + x$$

$$3\frac{1}{2} \text{ ft.} - 3 \text{ ft.} = 3 \text{ ft.} + x - 3 \text{ ft.}$$

$$\frac{1}{2} \text{ ft.} = x$$

There are three portions that make up the height of the flag. I know the lengths of two of them and the total. I can use this to solve for the length of the unknown piece.

The length of the missing piece is $\frac{1}{2}$ foot.

G6-M5-Lesson 11: Volume with Fractional Edge Lengths and Unit Cubes

1. Answer the following questions using this rectangular prism.

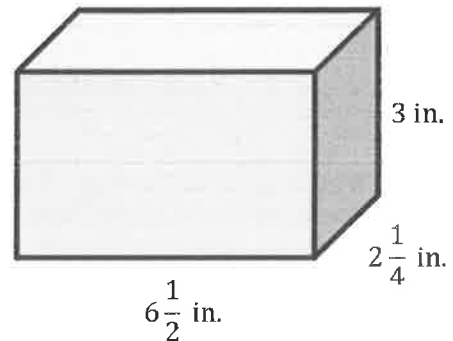
a. What is the volume of the prism?

$$V = lwh$$

$$V = \left(6\frac{1}{2} \text{ in.}\right) \left(2\frac{1}{4} \text{ in.}\right) (3 \text{ in.})$$

$$V = 43\frac{7}{8} \text{ in}^3$$

The volume of the prism is $43\frac{7}{8}$ cubic inches.



b. How many cubes with side lengths of $\frac{1}{4}$ in. would fill the prism?

$$\text{Cubes that fit along the length: } 6\frac{1}{2} = \frac{13}{2} = \frac{26}{4}$$

$$\text{Cubes that fit along the width: } 2\frac{1}{4} = \frac{9}{4}$$

$$\text{Cubes that fit along the height: } 3 = \frac{12}{4}$$

$$26 \times 9 \times 12 = 2,808$$

The prism could be filled with 2,808 cubes with a side length of $\frac{1}{4}$ in.

I can determine how many cubes fit along the length by finding equivalent fractions.

I need to convert the units to fourths to find how many $\frac{1}{4}$ -inch cubes fit along the length. $\frac{26}{4} = 26$ fourths, so I know 26 $\frac{1}{4}$ -inch cubes fit along the length.

c. How many cubes with side lengths of $\frac{1}{8}$ in. would fill the prism?

$$\text{Number of cubes} = 2(26) \times 2(9) \times 2(12)$$

$$= 52 \times 18 \times 24$$

$$= 22,464$$

The prism could be filled with 22,464 cubes with a side length of $\frac{1}{8}$ in.

I know that there are two $\frac{1}{8}$ inches in every $\frac{1}{4}$ inch, which means that there will be twice the amount of cubes for each dimension.

2. A company is packaging its décor for living rooms to be shipped to stores. Some of the décor is placed inside a cube-shaped package with side lengths of $4\frac{1}{3}$ in. These packages are then placed into a shipping box with dimensions of 26 in. \times 13 in. \times $8\frac{2}{3}$ in.

- a. What is the maximum number of smaller packages that can be placed into the larger shipping box?

$$26 \div 4\frac{1}{3} = \frac{26}{1} \div \frac{13}{3} = \frac{26}{1} \times \frac{3}{13} = 6$$

$$13 \div 4\frac{1}{3} = \frac{13}{1} \div \frac{13}{3} = \frac{13}{1} \times \frac{3}{13} = 3$$

$$8\frac{2}{3} \div 4\frac{1}{3} = \frac{26}{3} \div \frac{13}{3} = \frac{26}{3} \times \frac{3}{13} = 2$$

$$6 \times 3 \times 2 = 36$$

36 smaller packages can fit in the larger shipping box.

I need to determine how many times $4\frac{1}{3}$ in. will fit into each dimension.

- b. Use the number of packages that can be shipped in the box to help determine the volume of the shipping box.

$$4\frac{1}{3} \text{ in.} \times 4\frac{1}{3} \text{ in.} \times 4\frac{1}{3} \text{ in.}$$

$$\frac{13}{3} \text{ in.} \times \frac{13}{3} \text{ in.} \times \frac{13}{3} \text{ in.}$$

$$\frac{2,197}{27} \text{ in}^3$$

$$81\frac{10}{27} \text{ in}^3$$

One small package would have a volume of $81\frac{10}{27} \text{ in}^3$.

I can change the mixed numbers to fractions greater than one before I multiply.

$$36 \times 81\frac{10}{27} \text{ in}^3$$

$$\frac{36}{1} \times \frac{2,197}{27} \text{ in}^3$$

$$\frac{79,092}{27} \text{ in}^3$$

$$2,929\frac{1}{3} \text{ in}^3$$

The volume of the shipping box is $2,929\frac{1}{3} \text{ in}^3$.

I multiply the number of cubes by the volume of the cube.

3. A rectangular prism has a volume of 141.599 cubic centimeters. The height of the box is 8.9 centimeters, and the width is 3.7 centimeters.
- a. Write an equation that relates the volume to the length, width, and height. Let l represent the length, in centimeters.

$$141.599 = l(3.7)(8.9)$$

I can start with the formula $V = lwh$ and then fill in what I know.

- b. Solve the equation.

$$\begin{aligned}141.599 &= 32.93l \\141.599 \div 32.93 &= 32.93l \div 32.93 \\4.3 &= l\end{aligned}$$

This looks a lot like the equations I worked with in Module 4. I can use inverse operations to solve.

The length is 4.3 cm.

G6-M5-Lesson 12: From Unit Cubes to the Formulas for Volume

1. The area of the base of a rectangular prism is $7\frac{1}{2} \text{ m}^2$, and the height is $3\frac{3}{4} \text{ m}$. Determine the volume of the rectangular prism.

$$V = \text{Area of base} \times \text{height}$$

$$V = \left(7\frac{1}{2} \text{ m}^2\right) \left(3\frac{3}{4} \text{ m}\right)$$

$$V = \left(\frac{15}{2} \text{ m}^2\right) \left(\frac{15}{4} \text{ m}\right)$$

$$V = \frac{225}{8} \text{ m}^3$$

$$V = 28\frac{1}{8} \text{ m}^3$$

I was given the area of the base and the height. So instead of using the formula $V = lwh$, I will multiply the area of the base times the height.

2. The length of a rectangular prism is $4\frac{1}{2}$ times as long as the height. The width is $\frac{2}{3}$ of the height. The height is 12 cm. Determine the volume of the rectangular prism.

$$\text{Height} = 12 \text{ cm}$$

$$\text{Length} = 12 \text{ cm} \times 4\frac{1}{2} = 54 \text{ cm}$$

$$\text{Width} = 12 \text{ cm} \times \frac{2}{3} = 8 \text{ cm}$$

$$V = lwh$$

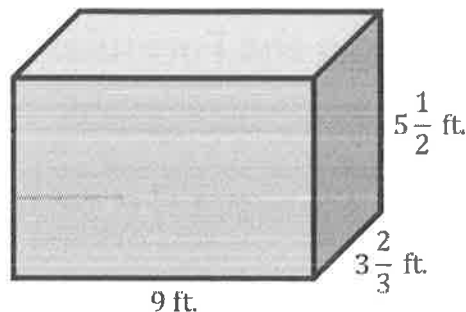
$$V = (54 \text{ cm})(8 \text{ cm})(12 \text{ cm})$$

$$V = 5,184 \text{ cm}^3$$

The volume of the rectangular prism is 5,184 cm³.

Before I can calculate the volume, I need to determine the length and width of the prism, and then I can use these measurements in the formula.

3.



I could find the volume by multiplying the length, width, and height. Or, I could find the area of the base first and then multiply this with the height.

- a. Write numerical expressions to represent the volume in two different ways, and explain what each reveals.

The first expression shows how I can find the volume using the formula $V = l w h$.

$$(9 \text{ ft.}) \left(3 \frac{2}{3} \text{ ft.}\right) \left(5 \frac{1}{2} \text{ ft.}\right)$$

The second expression shows how I can find the volume using the formula $V = B h$.

$$(33 \text{ ft}^2) \left(5 \frac{1}{2} \text{ ft.}\right)$$

- b. Determine the volume of the rectangular prism.

I can use either one of the expressions because they are equivalent expressions.

$$(9 \text{ ft.}) \left(3 \frac{2}{3} \text{ ft.}\right) \left(5 \frac{1}{2} \text{ ft.}\right) = 181 \frac{1}{2} \text{ ft}^3$$

OR

$$(33 \text{ ft}^2) \left(5 \frac{1}{2} \text{ ft.}\right) = 181 \frac{1}{2} \text{ ft}^3$$

4. The area of the base in this rectangular prism is fixed at 45 m^2 . This means that for the varying heights, there will be various volumes.

This means that the length and width will stay the same and only the height changes. Since the height is changing, the volume will change.

- a. Complete the table of values to determine the various heights and volumes.

I know the area of the base, so I can multiply what I know by the height to determine the volume.

| Height in Meters | Volume in Cubic Meters |
|------------------|------------------------|
| 1 | 45 |
| 2 | 90 |
| 3 | 135 |
| 4 | 180 |
| 5 | 225 |
| 6 | 270 |

- b. Write an equation to represent the relationship in the table. Be sure to define the variables used in the equation.

Let x represent the height of the rectangular prism in meters.

Let y represent the volume of the rectangular prism in cubic meters.

$$y = 45x$$

I know that the volume is equal to the area of the base times the height.

- c. What is the unit rate for this relationship? What does it mean in this situation?

The unit rate is 45.

For every meter of height, the volume increases by 45 m^3 because the area of the base is 45 m^2 . In order to determine the volume, multiply the height by 45 m^2 .

G6-M5-Lesson 13: The Formulas for Volume

1.

- a. Determine the volume of a cube with a side length of $3\frac{3}{4}$ cm.

$$V = lwh$$

$$V = \left(3\frac{3}{4} \text{ cm}\right)\left(3\frac{3}{4} \text{ cm}\right)\left(3\frac{3}{4} \text{ cm}\right)$$

I know that if the prism is a cube, the length, width, and height all have the same measure.

$$V = \left(\frac{15}{4} \text{ cm}\right)\left(\frac{15}{4} \text{ cm}\right)\left(\frac{15}{4} \text{ cm}\right)$$

$$V = \frac{3,375}{64} \text{ cm}^3$$

The volume of the cube is $\frac{3,375}{64} \text{ cm}^3$.

- b. Determine the volume of the cube in part (a) if all of the side lengths are doubled.

$$3\frac{3}{4} \text{ cm} \times 2 = 7\frac{1}{2} \text{ cm}$$

I must multiply the original dimensions by 2 to determine the dimensions of the new cube.

$$V = \left(7\frac{1}{2} \text{ cm}\right)\left(7\frac{1}{2} \text{ cm}\right)\left(7\frac{1}{2} \text{ cm}\right)$$

$$V = \left(\frac{15}{2} \text{ cm}\right)\left(\frac{15}{2} \text{ cm}\right)\left(\frac{15}{2} \text{ cm}\right)$$

$$V = \frac{3,375}{8} \text{ cm}^3$$

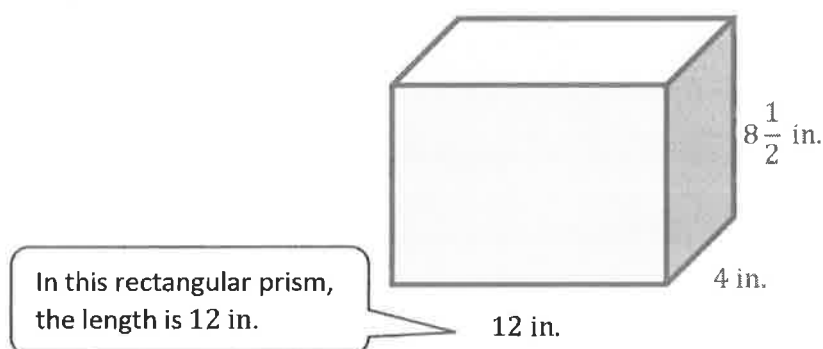
The volume of this cube is $\frac{3,375}{8} \text{ cm}^3$.

- c. Determine the relationship between the volumes in part (a) and part (b).

The volume of the prism in part (b) is 8 times greater than the volume of the prism in part (a).

I know the length is twice as long, the width is twice as long, and the height is twice as long as the original cube.
 $V = (2l)(2w)(2h) = 8lwh$

2. Use the rectangular prism to answer the following questions.



- a. Complete the table.

| Length | Volume |
|------------------------|-------------------------|
| $l = 12$ in. | 408 in ³ |
| $\frac{1}{2}l = 6$ in. | 204 in ³ |
| $\frac{1}{3}l = 4$ in. | 136 in ³ |
| $\frac{1}{4}l = 3$ in. | 102 in ³ |
| $2l = 24$ in. | 816 in ³ |
| $3l = 36$ in. | $1,224$ in ³ |
| $4l = 48$ in. | $1,632$ in ³ |

When I calculate the volumes in the table, the length changes, but the width and height stay the same.

- b. How did the volume change when the length was one-fourth as long as the original length?

When the length is one-fourth as long as the original length, the volume is one-fourth as much as the original volume.

I need to compare the new volume to the original volume. I can find the quotient of 102 in^3 and 408 in^3 to determine the change in volumes.

- c. How did the volume change when the length was four times as long as the original length?

When the length is four times as long as the original length, the volume is four times as much as the original volume.

I know that the difference in the volumes can be shown using subtraction.

3. The difference between the volumes of two rectangular prisms, Box A and Box B, is 23.87 cm^3 . Box B has a volume of 34.69 cm^3 .

- a. Let A represent the volume of Box A in cubic centimeters. Write an equation that could be used to determine the volume of Box A.

$$A - 34.69 \text{ cm}^3 = 23.87 \text{ cm}^3$$

- b. Solve the equation to determine the volume of Box A.

$$\begin{aligned} A - 34.69 \text{ cm}^3 &= 23.87 \text{ cm}^3 \\ A - 34.69 \text{ cm}^3 + 34.69 \text{ cm}^3 &= 23.87 \text{ cm}^3 + 34.69 \text{ cm}^3 \\ A &= 58.56 \text{ cm}^3 \end{aligned}$$

The volume of Box A is 58.56 cm^3 .

- c. If the area of the base of Box A is 4 cm^2 , write an equation that could be used to determine the height of Box A. Let h represent the height of Box A in centimeters.

$$\begin{aligned} V &= Bh \\ 58.56 \text{ cm}^3 &= (4 \text{ cm}^2)h \end{aligned}$$

- d. Solve the equation to determine the height of Box A.

$$58.56 \text{ cm}^3 = (4 \text{ cm}^2)h$$

$$58.56 \text{ cm}^3 \div 4 \text{ cm}^2 = (4 \text{ cm}^2)h \div 4 \text{ cm}^2$$

$$14.64 \text{ cm} = h$$

The height of Box A is 14.64 cm.

G6-M5-Lesson 14: Volume in the Real World

1. The volume of a rectangular prism is $\frac{35}{24}$ cm^3 , and the height of the prism is $\frac{7}{4}$ cm. Determine the area of the base.

Area of base = volume \div height

$$\text{Area of base} = \frac{35}{24} \text{ cm}^3 \div \frac{7}{4} \text{ cm}$$

$$\text{Area of base} = \frac{35}{24} \text{ cm}^3 \div \frac{42}{24} \text{ cm}$$

$$\text{Area of base} = 35 \text{ cm}^3 \div 42 \text{ cm}$$

$$\text{Area of base} = \frac{35}{42} \text{ cm}^2$$

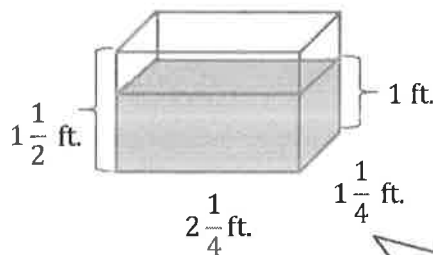
$$\text{Area of base} = \frac{5}{6} \text{ cm}^2$$

The area of the base is $\frac{5}{6}$ cm^2 .

I can use the equation
Volume = Area of base \times height and
then solve for the area of the base to
get a formula to use in this question.

I remember from Module 2 that I
can divide the numerators when I
have common denominators.

2. Determine the volume of the space in the tank that still needs to be filled with water.



Volume of tank = lwh

$$\text{Volume of tank} = \left(2\frac{1}{4} \text{ ft.}\right) \left(1\frac{1}{4} \text{ ft.}\right) \left(1\frac{1}{2} \text{ ft.}\right)$$

$$\text{Volume of tank} = \left(\frac{9}{4} \text{ ft.}\right) \left(\frac{5}{4} \text{ ft.}\right) \left(\frac{3}{2} \text{ ft.}\right)$$

$$\text{Volume of tank} = \frac{135}{32} \text{ ft}^3$$

I can determine the volume of
the tank and the volume of the
water that is already in the tank.
The difference between the two
will be the amount needed to fill
the rest of the tank.

Volume of water = lwh

$$\text{Volume of water} = \left(2\frac{1}{4} \text{ ft.}\right) \left(1\frac{1}{4} \text{ ft.}\right) (1 \text{ ft.})$$

$$\text{Volume of water} = \left(\frac{9}{4} \text{ ft.}\right) \left(\frac{5}{4} \text{ ft.}\right) (1 \text{ ft.})$$

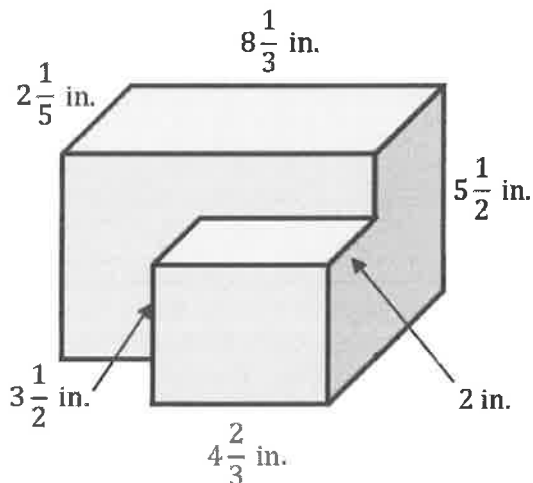
$$\text{Volume of water} = \frac{45}{16} \text{ ft}^3$$

Remaining water needed:

$$\frac{135}{32} \text{ ft}^3 - \frac{45}{16} \text{ ft}^3 = \frac{135}{32} \text{ ft}^3 - \frac{90}{32} \text{ ft}^3 = \frac{45}{32} \text{ ft}^3$$

The volume of the space that still needs to be filled is $\frac{45}{32} \text{ ft}^3$.

3.



This looks like two rectangular prisms that were stuck together.

I can find the volume of each piece and then add the volumes together.

a. Write an equation to represent the volume of the composite figure.

$$V = \left(8\frac{1}{3} \text{ in.} \times 2\frac{1}{5} \text{ in.} \times 5\frac{1}{2} \text{ in.}\right) + \left(4\frac{2}{3} \text{ in.} \times 2 \text{ in.} \times 3\frac{1}{2} \text{ in.}\right)$$

- b. Use your equation to calculate the volume of the composite figure.

$$V = \left(8\frac{1}{3} \text{ in.} \times 2\frac{1}{5} \text{ in.} \times 5\frac{1}{2} \text{ in.}\right) + \left(4\frac{2}{3} \text{ in.} \times 2 \text{ in.} \times 3\frac{1}{2} \text{ in.}\right)$$

$$V = \left(\frac{25}{3} \text{ in.} \times \frac{11}{5} \text{ in.} \times \frac{11}{2} \text{ in.}\right) + \left(\frac{14}{3} \text{ in.} \times \frac{2}{1} \text{ in.} \times \frac{7}{2} \text{ in.}\right)$$

$$V = \frac{605}{6} \text{ in}^3 + \frac{196}{6} \text{ in}^3$$

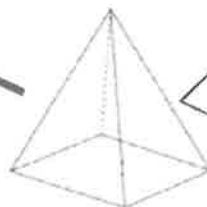
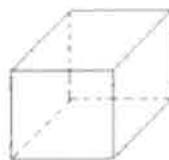
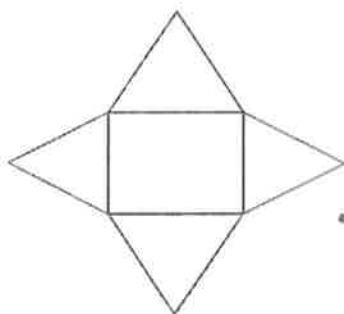
$$V = \frac{801}{6} \text{ in}^3$$

$$V = 133\frac{1}{2} \text{ in}^3$$

The volume of the composite figure is $133\frac{1}{2} \text{ in}^3$.

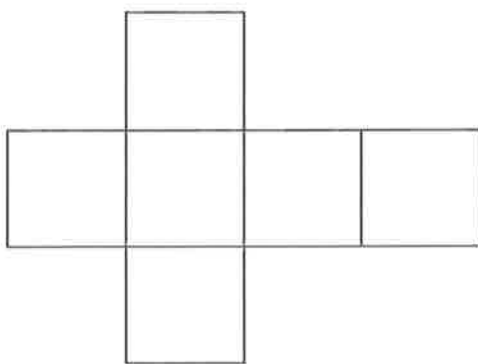
G6-M5-Lesson 15: Representing Three-Dimensional Figures Using Nets

1. Match the net below to the picture of its solid.

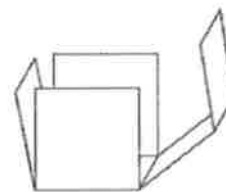


I know this shape is a pyramid because there are multiple identical triangles that form the lateral (side) faces of the pyramid, while the remaining face is the base (rectangle).

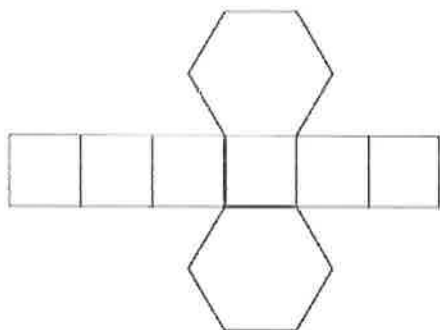
2. Sketch a net that can fold into a cube.



I know there are many possible nets for a cube. I can draw this net because I know a cube has six square faces. I can strategically draw the faces so the net, when folded along the edges, will create a closed, solid figure (in this case a cube). The image below shows what the net would look like when it's folded to create a cube.



3. Below is the net for a prism or pyramid. Classify the solid as prism or pyramid, and identify the shape of the base(s). Then, write the name of the solid.



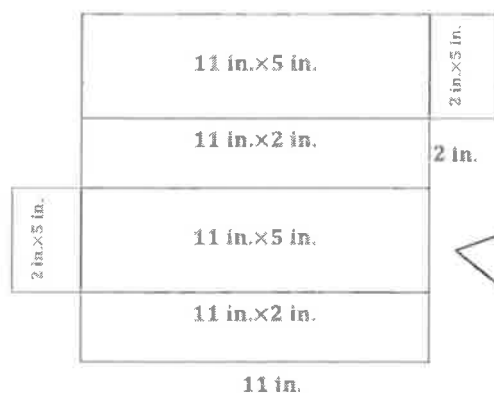
There are two identical bases, which are hexagons. Because the hexagon has six sides, there are six lateral faces, which are parallelograms. I remember a solid figure is named by the shape of its base.

This solid is a prism, and the bases are hexagons.

This is a hexagonal prism.

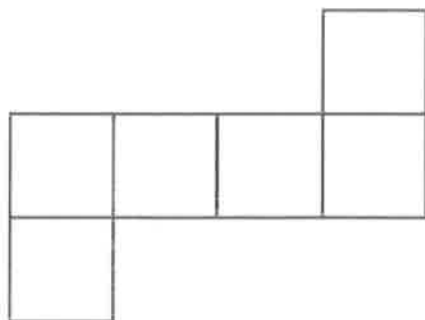
G6-M5-Lesson 16: Constructing Nets

1. Sketch and label the net of the following solid figures, and label the edge lengths.
- a. A granola cereal box that measures 11 inches high, 5 inches long, and 2 inches wide



I can use the dimensions of this rectangular prism (granola cereal box) to create the net. There are three pairs of faces with identical areas, and I can arrange them so that this net will fold into a rectangular prism. The dimensions help me label the edge lengths.

- b. A cubic box that measures 12 cm on each edge.
All edges are 12 cm.



The net of a cube has 6 identical square faces. This is one possible arrangement of the faces.

- c. Challenge: Write a numerical expression for the total area of the net in part (b). Tell what each of the terms in your expression means.

$$6(12 \text{ cm} \times 12 \text{ cm})$$

This is how I calculate the area of one face.

This is another way of saying 6 groups of $(12 \text{ cm} \times 12 \text{ cm})$ since there are 6 congruent faces and each face is a square with an edge of 12 cm.

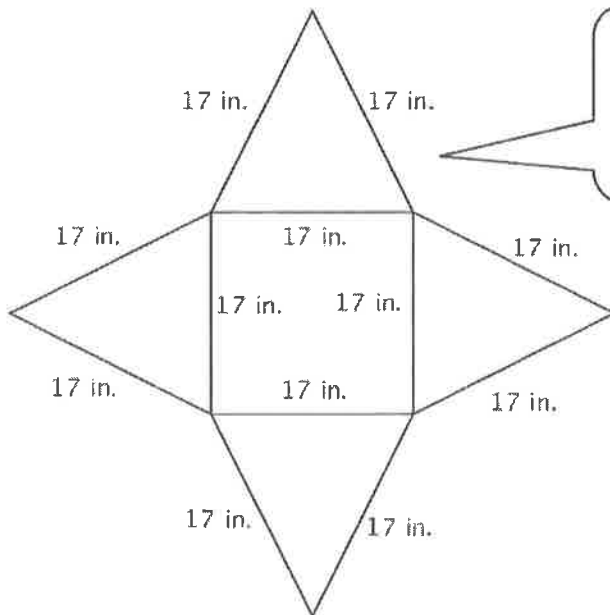
$$(12 \text{ cm} \times 12 \text{ cm}) + (12 \text{ cm} \times 12 \text{ cm}) + (12 \text{ cm} \times 12 \text{ cm}) + (12 \text{ cm} \times 12 \text{ cm}) + (12 \text{ cm} \times 12 \text{ cm}) + (12 \text{ cm} \times 12 \text{ cm})$$

I could also show 6 groups of $(12 \text{ cm} \times 12 \text{ cm})$ like this.

There are 6 faces in the cube, and each has dimensions 12 cm by 12 cm.

2. The base for a patio umbrella is shaped like a square pyramid. The pyramid has equilateral faces that measure 17 inches on each side. The base is 17 inches on each side. Sketch the net of the umbrella base, and label the edge lengths.

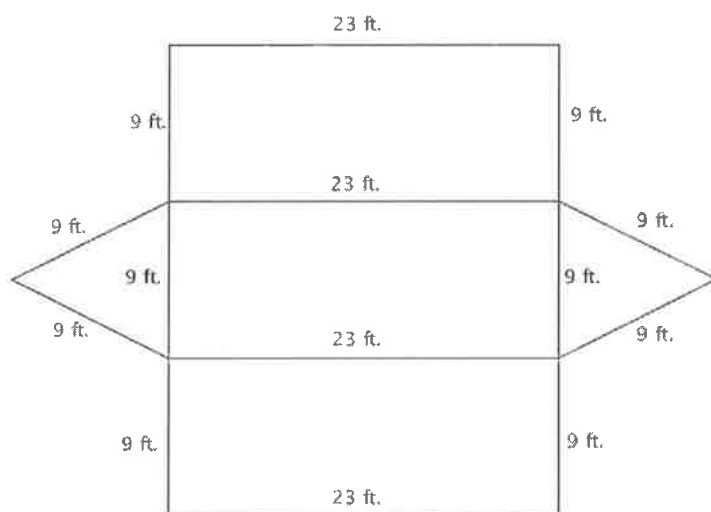
Possible Net:



Because this is a square pyramid, I know the base is a square (all edges are 17 in.) and all the lateral faces are triangles. Since the base has four sides, there will be four triangular faces.

3. The roof of a detached garage is in the shape of a triangular prism. It has equilateral bases that measure 9 feet on each side. The length of the roof is 23 feet. Sketch the net of the detached garage, and label the edge lengths.

Possible Net:

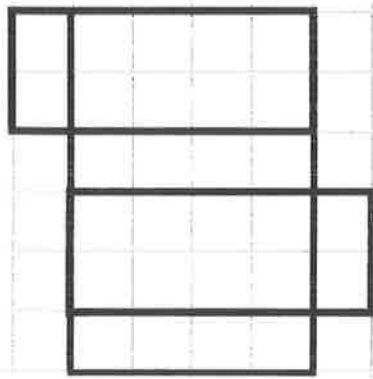


Since this is a triangular prism, I know there are two triangular bases. Since the base is a triangle and has three sides, there will be three rectangular faces.

G6-M5-Lesson 17: From Nets to Surface Area

Name the shape, and write an expression for surface area. Calculate the surface area of the figure. Assume each box on the grid paper represents a 1 ft. \times 1 ft. square.

1.



Name of Shape: Rectangular Prism

I know this is a rectangular prism because there are six rectangular faces.

To find the surface area, I can find the area of each face and then find the sum of the areas of all six faces.

Surface Area:

$$\begin{aligned} &(2 \text{ ft.} \times 1 \text{ ft.}) + (2 \text{ ft.} \times 1 \text{ ft.}) + (4 \text{ ft.} \times 2 \text{ ft.}) + (4 \text{ ft.} \times 2 \text{ ft.}) + (4 \text{ ft.} \times 1 \text{ ft.}) + (4 \text{ ft.} \times 1 \text{ ft.}) \\ &= 2(2 \text{ ft.} \times 1 \text{ ft.}) + 2(4 \text{ ft.} \times 2 \text{ ft.}) + 2(4 \text{ ft.} \times 1 \text{ ft.}) \\ &= 4 \text{ ft}^2 + 16 \text{ ft}^2 + 8 \text{ ft}^2 \\ &= 28 \text{ ft}^2 \end{aligned}$$

I notice there are three groups of faces that have identical areas, so I can rewrite the expression to reflect this idea.

Explain the error in Problems 2 and 3. Then, correct the error. Assume each box on the grid paper represents a 1 cm \times 1 cm square.

2. Name of Shape: Rectangular Prism or, more specifically, a Cube

$$\text{Area of Faces: } 2 \text{ cm} \times 2 \text{ cm} = 4 \text{ cm}^2$$

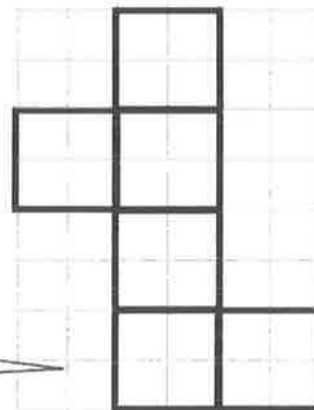
$$\text{Surface Area: } 4 \text{ cm}^2 + 4 \text{ cm}^2 + 4 \text{ cm}^2 + 4 \text{ cm}^2 = 16 \text{ cm}^2$$

The solution shown above only calculates the sum of four faces, but a cube has six faces so the solution above is incorrect.

$$4 \text{ cm}^2 + 4 \text{ cm}^2 + 4 \text{ cm}^2 + 4 \text{ cm}^2 + 4 \text{ cm}^2 + 4 \text{ cm}^2 = 24 \text{ cm}^2$$

Therefore, the correct surface area is 24 cm² and not 16 cm².

I see there are six faces when looking at the net, but the work for surface area only shows the sum of four faces.



3. Name of Shape: Rectangular Pyramid, but more specifically, a Square Pyramid

$$\text{Area of Base: } 4 \text{ m} \times 4 \text{ m} = 12 \text{ m}^2$$

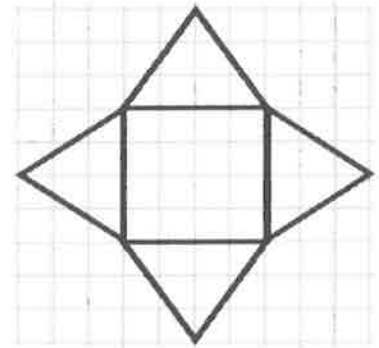
$$\text{Area of Triangles: } \frac{1}{2} \times 4 \text{ m} \times 3 \text{ m} = 6 \text{ m}^2$$

$$\text{Surface Area: } 12 \text{ m}^2 + 6 \text{ m}^2 + 6 \text{ m}^2 + 6 \text{ m}^2 + 6 \text{ m}^2 = 36 \text{ m}^2$$

The method used to calculate the surface area is correct, but there is a math error in the calculation of the area of the base. The area of the base should be 16 m^2 because the product of 4 m and 4 m is 16 m^2 .

$$16 \text{ m}^2 + 6 \text{ m}^2 + 6 \text{ m}^2 + 6 \text{ m}^2 + 6 \text{ m}^2 = 40 \text{ m}^2$$

Therefore, the correct surface area of the square pyramid is 40 m^2 .



4. Catherine and Julia are both writing expressions to calculate the surface area of a rectangular prism. However, they wrote different expressions.

- a. Examine the expressions below, and determine if they represent the same value. Explain why or why not.

Catherine's Expression

$$(6 \text{ in.} \times 9 \text{ in.}) + (6 \text{ in.} \times 9 \text{ in.}) + (6 \text{ in.} \times 10 \text{ in.}) + (6 \text{ in.} \times 10 \text{ in.}) + (9 \text{ in.} \times 10 \text{ in.}) + (9 \text{ in.} \times 10 \text{ in.})$$

Julia's Expression

$$2(6 \text{ in.} \times 9 \text{ in.}) + 2(6 \text{ in.} \times 10 \text{ in.}) + 2(9 \text{ in.} \times 10 \text{ in.})$$

Julia realized there are three groups of identical products, so she combined these using the distributive property.

The expressions are equivalent expressions because they have the same value, but Julia used the distributive property to write a more compact expression than Catherine's.

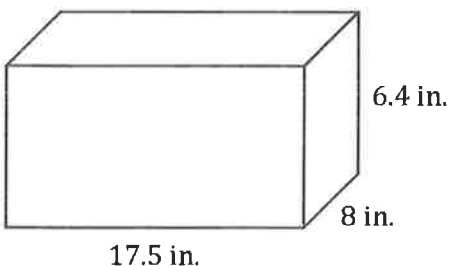
- b. What fact about the surface area of a rectangular prism does Julia's expression show that Catherine's does not?

A rectangular prism is composed of three pairs of sides with identical areas.

G6-M5-Lesson 18: Determining Surface Area of Three-Dimensional Figures

Calculate the surface area of the figures below. This figures are not drawn to scale.

1.



The formula $SA = 2(l \times w) + 2(l \times h) + 2(w \times h)$ can be used to determine the surface area of this figure.

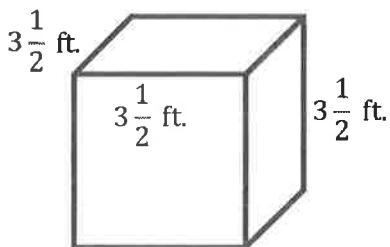
$$SA = 2(17.5 \text{ in.})(8 \text{ in.}) + 2(17.5 \text{ in.})(6.4 \text{ in.}) + 2(8 \text{ in.})(6.4 \text{ in.})$$

$$SA = 280 \text{ in}^2 + 224 \text{ in}^2 + 102.4 \text{ in}^2$$

$$SA = 606.4 \text{ in}^2$$

The surface area of the figure is 606.4 in^2 .

2.



$$SA = 6 \left(3 \frac{1}{2} \text{ ft.}\right)^2$$

$$SA = 6 \left(\frac{7}{2} \text{ ft.}\right)^2$$

$$SA = 6 \left(\frac{49}{4} \text{ ft}^2\right)$$

$$SA = \frac{294}{4} \text{ ft}^2$$

$$SA = 73 \frac{1}{2} \text{ ft}^2$$

All six faces of a cube are identical squares, which means I can square the side length since the side lengths are the same and then multiply by 6.

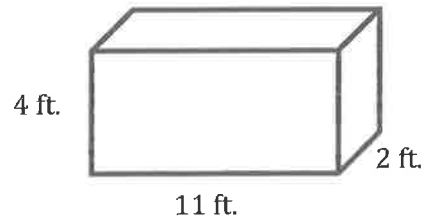
The surface area of the figure is $73 \frac{1}{2} \text{ ft}^2$.

3. Write a numerical expression to show how to calculate the surface area of the rectangular prism. Explain each part of the expression.

$$2(4 \text{ ft.} \times 11 \text{ ft.}) + 2(4 \text{ ft.} \times 2 \text{ ft.}) + 2(11 \text{ ft.} \times 2 \text{ ft.})$$

The first term of the expression shows the area of the front and back faces of the rectangular prism. The second term of the expression shows the area of the left and right faces of the rectangular prism. The third term shows the area of the top and bottom faces of the rectangular prism.

The surface area of the figure is 148 ft^2 .



4. When Annabelle was calculating the surface area for Problem 1, she identified the following:

$$\text{length} = 17.5 \text{ in.}, \text{ width} = 8 \text{ in.}, \text{ and height} = 6.4 \text{ in.}$$

However, when Vincent was calculating the surface area for the same problem, he identified the following:

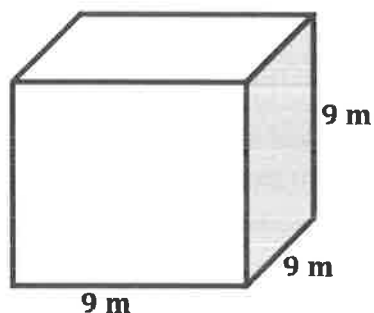
$$\text{length} = 8 \text{ in.}, \text{ width} = 17.5 \text{ in.}, \text{ and height} = 6.4 \text{ in.}$$

Would Annabelle and Vincent get the same answer? Why or why not?

Annabelle and Vincent would get the same answer because they are still finding the correct area of all six faces of the rectangular prism.

The surface area will be the same because the dimensions represent the same rectangular prism, but it may be in a different position.

5. Examine the figure below.



- a. What is the most specific name of the three-dimensional shape?

Cube

- b. Write two different expressions for the surface area.

$$(9\text{ m} \times 9\text{ m}) + (9\text{ m} \times 9\text{ m}) + (9\text{ m} \times 9\text{ m}) + (9\text{ m} \times 9\text{ m}) + (9\text{ m} \times 9\text{ m}) + (9\text{ m} \times 9\text{ m})$$

$$6(9\text{ m})^2$$

I know that a cube is a unique rectangular prism because it has 6 identical faces. I can use the formula for surface area of a rectangular prism or the formula $SA = 6s^2$, which is more efficient for a cube.

- c. Explain how these two expressions are equivalent.

The two expressions are equivalent because the first expression shows the sum of the areas of each face. The second expression is a more compact expression because each face has the same area and there are 6 groups of 9 m^2 .

G6-M5-Lesson 19: Surface Area and Volume in the Real World

1. Samuel built a small wooden box to hold nails. Each side of the box measures 7 inches.
- a. How many square inches of wood did he use to build the box?

Surface Area of the Box: $SA = 6(7 \text{ in.})^2 = 6(49 \text{ in}^2) = 294 \text{ in}^2$

Samuel used 294 square inches of wood to build the box.

I need to determine the surface area because it measures the total area of the surface of a figure.

- b. How many cubic inches of nails does the box hold?

Volume of the Box: $V = 7 \text{ in.} \times 7 \text{ in.} \times 7 \text{ in.} = 343 \text{ in}^3$

The box holds 343 cubic inches of nails.

The volume of a cube measures the space inside a three dimensional figure. To calculate the volume of a cube, I multiply the length, width, and height, or $V = s^3$ since the dimensions are the same.

2. A company that manufactures containers wants to know how many different containers it can make if the dimensions must be whole numbers and each container has a volume of 18 cubic centimeters.
- a. List all the possible whole number dimensions for the box.

Choice One: $1 \text{ cm} \times 1 \text{ cm} \times 18 \text{ cm}$

Choice Two: $1 \text{ cm} \times 2 \text{ cm} \times 9 \text{ cm}$

Choice Three: $1 \text{ cm} \times 3 \text{ cm} \times 6 \text{ cm}$

Choice Four: $2 \text{ cm} \times 3 \text{ cm} \times 3 \text{ cm}$

I know the dimensions of each container are factors of 18.

- b. Which possibility requires the least amount of material to make?

Choice One:

$$SA = 2(1 \text{ cm})(1 \text{ cm}) + 2(1 \text{ cm})(18 \text{ cm}) + 2(1 \text{ cm})(18 \text{ cm})$$

$$SA = 2 \text{ cm}^2 + 36 \text{ cm}^2 + 36 \text{ cm}^2$$

$$SA = 74 \text{ cm}^2$$

I need to calculate the surface area of each container to determine the one that requires the least amount of material.

Choice Two:

$$SA = 2(1 \text{ cm})(2 \text{ cm}) + 2(1 \text{ cm})(9 \text{ cm}) + 2(2 \text{ cm})(9 \text{ cm})$$

$$SA = 4 \text{ cm}^2 + 18 \text{ cm}^2 + 36 \text{ cm}^2$$

$$SA = 58 \text{ cm}^2$$

Choice Three:

$$SA = 2(1 \text{ cm})(3 \text{ cm}) + 2(1 \text{ cm})(6 \text{ cm}) + 2(3 \text{ cm})(6 \text{ cm})$$

$$SA = 6 \text{ cm}^2 + 12 \text{ cm}^2 + 36 \text{ cm}^2$$

$$SA = 54 \text{ cm}^2$$

Choice Four:

$$SA = 2(2 \text{ cm})(3 \text{ cm}) + 2(2 \text{ cm})(3 \text{ cm}) + 2(3 \text{ cm})(3 \text{ cm})$$

$$SA = 12 \text{ cm}^2 + 12 \text{ cm}^2 + 18 \text{ cm}^2$$

$$SA = 42 \text{ cm}^2$$

Choice Four requires the least amount of material because it has the smallest surface area.

- c. Which box would you recommend the company use? Why?

I would recommend the box with dimensions of 2 cm × 3 cm × 3 cm (Choice Four) because it requires the least amount of material to make, which means it costs the company the least amount of money to make.

3. Auntie Math, Co. has two different boxes for Auntie Math Cereal. The large box is 7.5 inches wide, 8 inches high, and 3 inches deep. The small box is 4 inches wide, 11 inches high, and 1.5 inches deep.
- a. How much more cardboard is needed to make the large box than the small box?

Surface Area of the Large Box:

$$\begin{aligned} &2(7.5 \text{ in.})(8 \text{ in.}) + 2(7.5 \text{ in.})(3 \text{ in.}) + 2(8 \text{ in.})(3 \text{ in.}) \\ &120 \text{ in}^2 + 45 \text{ in}^2 + 48 \text{ in}^2 \\ &213 \text{ in}^2 \end{aligned}$$

Before I can answer the question, I need to calculate the surface area for each box.

Surface Area of the Small Box:

$$\begin{aligned} &2(4 \text{ in.})(11 \text{ in.}) + 2(4 \text{ in.})(1.5 \text{ in.}) + 2(11 \text{ in.})(1.5 \text{ in.}) \\ &88 \text{ in}^2 + 12 \text{ in}^2 + 33 \text{ in}^2 \\ &133 \text{ in}^2 \end{aligned}$$

Difference:

$$\begin{aligned} &213 \text{ in}^2 - 133 \text{ in}^2 \\ &80 \text{ in}^2 \end{aligned}$$

The large box requires 80 square inches more material than the small box.

- b. How much more cereal does the large box hold than the small box?

Volume of the Large Box:

$$\begin{aligned} &7.5 \text{ in.} \times 8 \text{ in.} \times 3 \text{ in.} \\ &180 \text{ in}^3 \end{aligned}$$

Before answering the question, I need to calculate the volume of each box.

Volume of the Small Box:

$$\begin{aligned} &4 \text{ in.} \times 11 \text{ in.} \times 1.5 \text{ in.} \\ &66 \text{ in}^3 \end{aligned}$$

Difference:

$$\begin{aligned} &180 \text{ in}^3 - 66 \text{ in}^3 \\ &114 \text{ in}^3 \end{aligned}$$

The large box holds 114 cubic inches more cereal than the small box.

4. A swimming pool is 9 meters long, 5 meters wide, and 2 meters deep. The water-resistant paint needed for the pool costs \$5 per square meter. How much will it cost to paint the pool?

- a. How many faces of the pool do you have to paint?

You have to paint 5 faces.

I do not have to paint the top of the pool.

- b. How much paint (in square meters) do you need to paint the pool?

Surface area of all six faces:

$$2(9 \text{ m} \times 5 \text{ m}) + 2(9 \text{ m} \times 2 \text{ m}) + 2(5 \text{ m} \times 2 \text{ m})$$

$$90 \text{ m}^2 + 36 \text{ m}^2 + 20 \text{ m}^2$$

$$146 \text{ m}^2$$

Area of Top of Pool:

$$9 \text{ m} \times 5 \text{ m}$$

$$45 \text{ m}^2$$

Total Paint Needed:

$$146 \text{ m}^2 - 45 \text{ m}^2$$

$$101 \text{ m}^2$$

I can also use the expression $9 \text{ m} \times 5 \text{ m} + 2(9 \text{ m} \times 2 \text{ m}) + 2(5 \text{ m} \times 2 \text{ m})$ since there is only ONE face with the dimensions $9 \text{ m} \times 5 \text{ m}$.

- c. How much will it cost to paint the pool?

$$101 \times 5 = 505$$

It will cost \$505 to paint the pool.

5. The volume of Box X subtracted from the volume of Box Y is 16.2 cubic centimeters. Box X has a volume of 5.63 cubic centimeters.

- a. Let Y be the volume of Box Y in cubic centimeters. Write an equation that could be used to determine the volume of Box X.

$$Y - 5.63 \text{ cm}^3 = 16.2 \text{ cm}^3$$

Let X be the volume of box X. I know $Y - X$ is 16.2 cm^3 , so I can substitute the volume of Box X in for X .

- b. Solve the equation to determine the volume of Box Y.

$$Y - 5.63 \text{ cm}^3 = 16.2 \text{ cm}^3$$

$$Y - 5.63 \text{ cm}^3 + 5.63 \text{ cm}^3 = 16.2 \text{ cm}^3 + 5.63 \text{ cm}^3$$

$$Y = 21.83 \text{ cm}^3$$

I have a lot of experience with isolating the variable in an equation.

- c. The volume of Box Y is one-tenth the volume of another box, Box Z. Let Z represent the volume of Box Z. Write an equation that could be used to determine the volume of Box Z, using the result from part (b).

$$21.83 \text{ cm}^3 = \frac{1}{10}Z$$

- d. Solve the equation to determine the volume of Box Z.

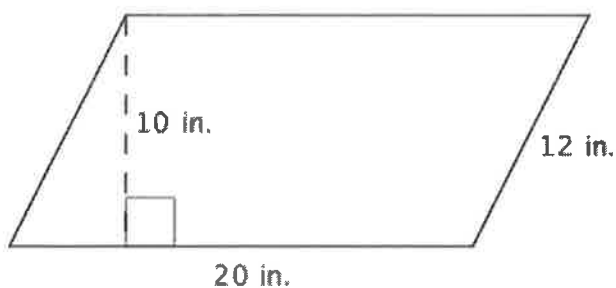
$$21.83 \text{ cm}^3 = \frac{1}{10}Z$$

$$21.83 \text{ cm}^3 \div \frac{1}{10} = \frac{1}{10}Z \div \frac{1}{10}$$

$$218.3 \text{ cm}^3 = Z$$

G6-M5-Lesson 19a: Applying Surface Area and Volume to Aquariums

1. Calculate the area of the figure below.



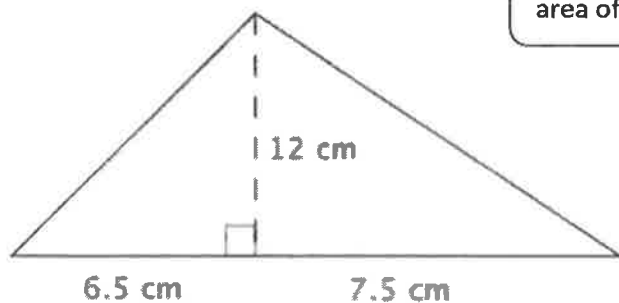
I can use the formula for area of a parallelogram.

$$A = bh$$

$$A = (20 \text{ in.})(10 \text{ in.})$$

$$A = 200 \text{ in}^2$$

2. Calculate the area of the figure below.



I can use the formula for area of a triangle.

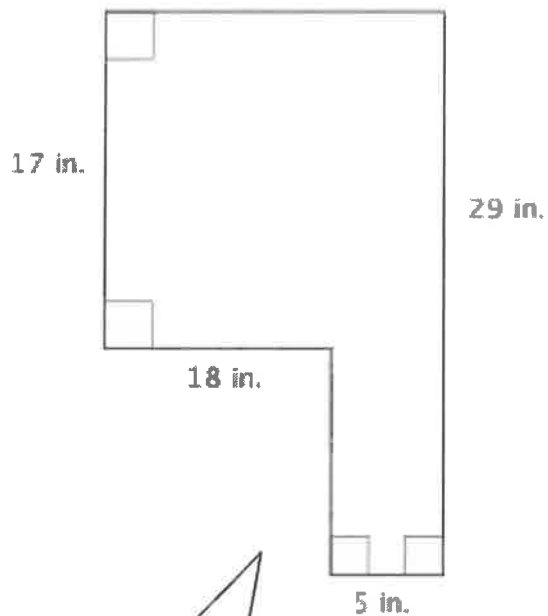
$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}(14 \text{ cm})(12 \text{ cm})$$

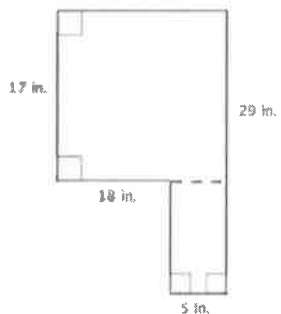
$$A = 84 \text{ cm}^2$$

The base of the triangle is 14 cm because 14 cm is the sum of 6.5 cm and 7.5 cm.

3. Calculate the area of the figure below.



I can decompose this figure into two rectangles.



Area of top rectangle:

$$A = lw$$

$$A = (23 \text{ in.})(17 \text{ in.})$$

$$A = 391 \text{ in}^2$$

Area of bottom rectangle:

$$A = lw$$

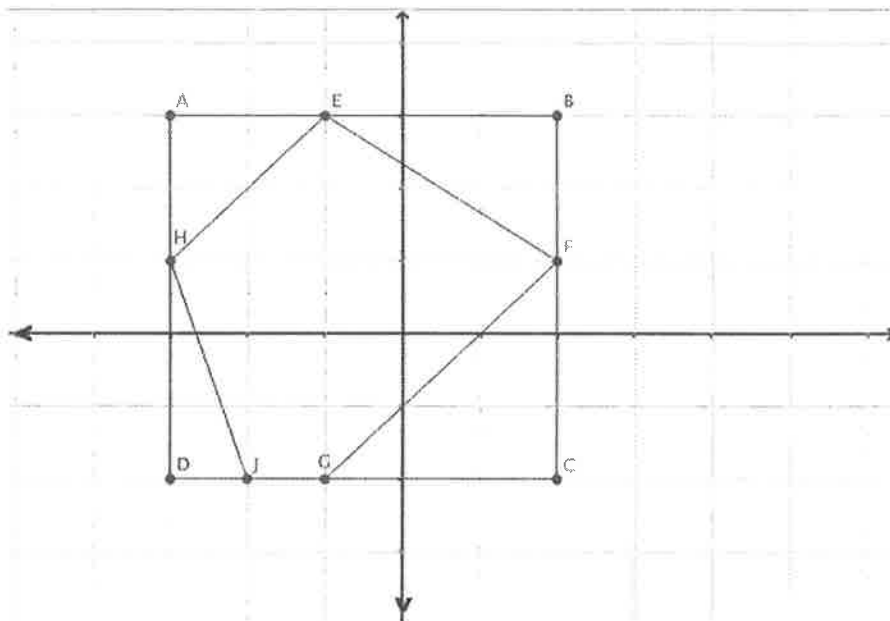
$$A = (5 \text{ in.})(12 \text{ in.})$$

$$A = 60 \text{ in}^2$$

I determine the width of this rectangle by subtracting 17 in. from 29 in.

$$29 \text{ in.} - 17 \text{ in.} = 12 \text{ in.}$$

4. Complete the table using the diagram on the coordinate plane to find the distance between the two points on each segment.

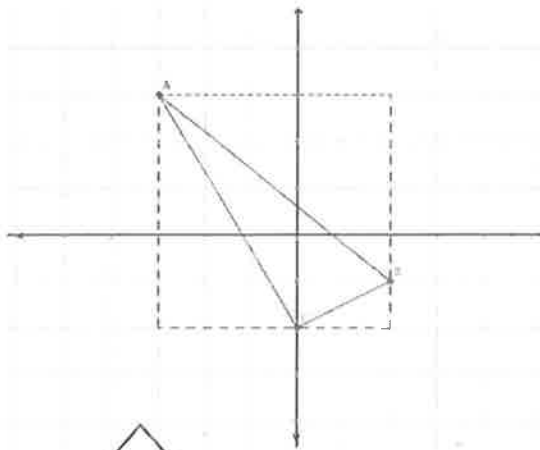


| Line Segment | Point | Point | Distance | Proof |
|-----------------|------------|------------|----------|------------------|
| \overline{AB} | $(-3, 3)$ | $(2, 3)$ | 5 | $ -3 + 2 = 5$ |
| \overline{BF} | $(2, 3)$ | $(2, 1)$ | 2 | $ 3 - 1 = 2$ |
| \overline{CG} | $(2, -2)$ | $(-1, -2)$ | 3 | $ 2 + -1 = 3$ |
| \overline{DH} | $(-3, -2)$ | $(-3, 1)$ | 3 | $ -2 + 1 = 3$ |
| \overline{HA} | $(-3, 1)$ | $(-3, 3)$ | 2 | $ 3 - 1 = 2$ |
| \overline{AD} | $(-3, 3)$ | $(-3, -2)$ | 5 | $ 3 + -2 = 5$ |

I find the location of each point in the line segment on the coordinate plane and record the coordinates of each point. If the points are on the same side of 0, I subtract the absolute values of the different coordinate to find the distance between the two points. If the points are on opposite sides of 0, I add the absolute values of the coordinates to find the distance between the two points.

5. Plot the points below, and draw the shape. Then, determine the area of the polygon.

$$A(-3, 3), B(2, -1), C(0, -2)$$



I can plot each point on the coordinate plane. To find the area of the polygon, I can draw a rectangle around the figure and find the area of the sections around the polygon. Since each of the surrounding sections is a triangle, I use the formula for area of a triangle to find the area of each section. Then, to find the area of the polygon, I can subtract the area of each section

Area of rectangle:

$$\text{Area} = lw$$

$$\text{Area} = (5 \text{ units})(5 \text{ units})$$

$$\text{Area} = 25 \text{ units}^2$$

Area on left:

$$\text{Area} = \frac{1}{2}bh$$

$$\text{Area} = \frac{1}{2}(3 \text{ units})(5 \text{ units})$$

$$\text{Area} = 7.5 \text{ units}^2$$

Area on top:

$$\text{Area} = \frac{1}{2}bh$$

$$\text{Area} = \frac{1}{2}(5 \text{ units})(4 \text{ units})$$

$$\text{Area} = 10 \text{ units}^2$$

Area on right:

$$\text{Area} = \frac{1}{2}bh$$

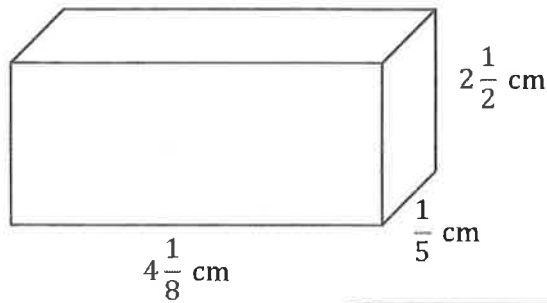
$$\text{Area} = \frac{1}{2}(2 \text{ units})(1 \text{ units})$$

$$\text{Area} = 1 \text{ unit}^2$$

$$\text{Area of the Polygon} = 25 \text{ units}^2 - 7.5 \text{ units}^2 - 10 \text{ units}^2 - 1 \text{ unit}^2$$

$$\text{Area of the Polygon} = 6.5 \text{ units}^2$$

6. Determine the volume of the figure.



$$V = lwh$$

$$V = \left(4\frac{1}{8} \text{ cm}\right) \left(\frac{1}{5} \text{ cm}\right) \left(2\frac{1}{2} \text{ cm}\right)$$

$$V = \frac{165}{80} \text{ cm}^3$$

$$V = 2\frac{5}{80} \text{ cm}^3 = 2\frac{1}{16} \text{ cm}^3$$

I can rename the mixed numbers as fractions greater than one to multiply more efficiently.

Now, I can rename the fraction greater than one as a mixed number. Since $\frac{80}{80}$ is 1, $\frac{160}{80}$ is 2 and $\frac{5}{80}$ remain. So, I can rename $\frac{165}{80}$ as $2\frac{5}{80}$, which can be renamed as $2\frac{1}{16}$.

7. Give at least three more expressions that could be used to determine the volume of the figure in Problem 6.

Answers may vary. Some examples include the following.

In this expression, I can rearrange the length, width, and height and then multiply.

$$\left(\frac{1}{5} \text{ cm}\right) \left(2\frac{1}{2} \text{ cm}\right) \left(4\frac{1}{8} \text{ cm}\right)$$

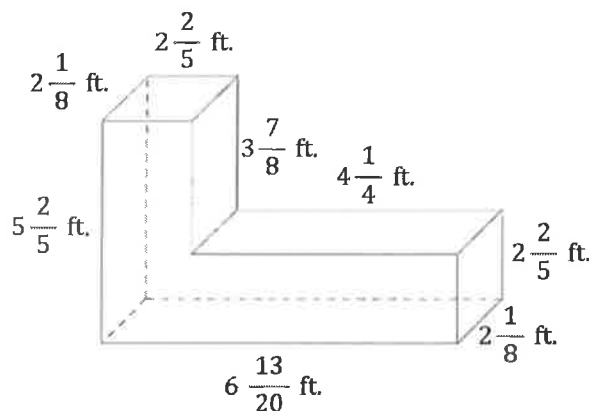
$$\left(\frac{33}{40} \text{ cm}^2\right) \left(2\frac{1}{2} \text{ cm}\right)$$

$$\left(\frac{5}{10} \text{ cm}^2\right) \left(4\frac{1}{8} \text{ cm}\right)$$

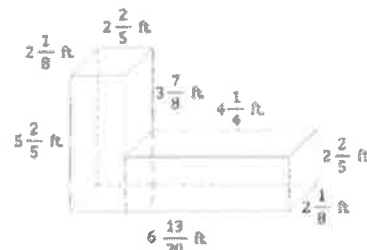
Here, I can calculate the area of the base ($l \times w$) and then multiply by the height.

Because of the commutative property, I can multiply the width by the height ($w \times h$) and then multiply the product by the length.

8. Determine the volume of the irregular figure.



I can cut the figure into two parts. Each part is a rectangular prism. Then, I can find the volume of each rectangular prism and add the volumes together.



Volume of the right Rectangular Prism:

$$\begin{aligned}
 V &= lwh \\
 V &= \left(4\frac{1}{4}\text{ ft.}\right)\left(2\frac{2}{5}\text{ ft.}\right)\left(2\frac{1}{8}\text{ ft.}\right) \\
 V &= \frac{3468}{160}\text{ ft}^3 \\
 V &= 21\frac{108}{160}\text{ ft}^3 = 21\frac{27}{40}\text{ ft}^3
 \end{aligned}$$

Volume of the left Rectangular Prism:

$$\begin{aligned}
 V &= lwh \\
 V &= \left(5\frac{2}{5}\text{ ft.}\right)\left(2\frac{1}{8}\text{ ft.}\right)\left(2\frac{2}{5}\text{ ft.}\right) \\
 V &= \frac{5,508}{200}\text{ ft}^3 \\
 V &= 27\frac{108}{200}\text{ ft}^3 = 27\frac{27}{50}\text{ ft}^3
 \end{aligned}$$

$$\text{Total Volume} = 21\frac{27}{40}\text{ ft}^3 + 27\frac{27}{50}\text{ ft}^3 = 49\frac{43}{200}\text{ ft}^3$$

